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Voltage Stability-Constrained Allocation of EV Parking Lots in the Distribution System

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Abstract
With the large number of electric vehicles (EV) in charging stations, the power system will face a large amount of charging demand. It can lead to voltage instability and higher power loss in the electricity distribution network. However, by constructing EV charging parking lots in suitable places and managing them optimally, we can benefit from the advantage of using the battery capacity of EVs. This article proposes a mixed-integer linear programming model for locating and sizing electric vehicle parking lots (EVPL) to maximize the profit of the EVPL owner, taking into account the network constraints and the voltage stability index (VSI). Moreover, the impact of different travel patterns for EVs on working days and weekends has also been investigated. The desired model has been implemented in a distribution system with 37 buses, which includes four different areas regarding the type of travel. The results show that the VSI drops in the presence of EVPLs. However, it can be constrained through optimal location and management of EVPLs.

Keywords
Distribution system, Electric Vehicle, Allocation, Parking Lot, Voltage Stability.

1. Introduction

The increasing penetration of electric vehicles (EV) in urban transportation systems creates challenges in the distribution network (DN). The increase in load caused by EV charging has adverse effects, such as reducing voltage stability, increasing network losses, and reducing system reliability. Therefore, the network operators are looking to provide the necessary charging infrastructure, such as electric vehicle parking lots (EVPL), and improve the system condition with proper charging management [1-3]. Therefore, determining the location and capacity of EVPLs is becoming an important issue. Usually, EVs are parked for a long time during the day. Therefore, EVPLs can employ vehicle-to-grid (V2G) technology and act as energy storage to tackle the challenges caused by their presence in the DN [2-4]. However, locating parking lots is a complex issue that should be considered comprehensively and from different perspectives, such as transportation networks, distribution networks, EV owners, and EVPL owners. So far, much research has been proposed to determine the optimal location and size of EVPLs. In [1], a multi-objective optimization model has been formulated to find the optimal location of EVPLs and reinforcement of the DN, regarding the requirements of EV owners, EVPL investors, and distribution system operators (DSO). Some researchers have proposed a two-step method for allocating EVPLs [5], or EVPLs and distributed generations (DG) [6] in the DN. The planning objective is optimized in the first stage, while the operation decisions such as minimizing power loss or voltage deviation are made in the second stage. In [7,8], the charging and discharging of EVs have been managed in two stages to maximize the profits of the EV owners and the charging station (CS) operator. The output of the first stage is the optimal CS demand. Then, the optimal location of the CS is determined to minimize power loss and voltage deviation, while maximizing the voltage stability index (VSI) in the second stage. However, the proposed model is non-linear because of the index defined for voltage stability. Reference [9] considers user behavior uncertainty in a two-stage stochastic programming model. The first stage deals with planning decisions on the location and size of PLs. In the second stage, the performance of the proposed PL system is evaluated under the realization of different scenarios of EV owners' behavior. In [10], a dynamic planning method is presented for optimally determining the location, capacity, and time of construction and development of EVPLs to minimize the time and energy required to reach the stations. Other articles also discuss the location and capacity of EVPLs with objectives such as reducing losses [6,11,12], improving reliability [12], maximizing the profit of PLs combined with reducing losses and increasing reliability [13, 14], minimizing bus voltage deviation [5, 7, 15, 16], increasing the welfare of EV owners [2,17] and maximizing the profit of the DSO [2, 18]. However, the charging pattern of EVs has not been optimized in [6, 12]. In [11, 14], the charging pattern and the optimal location of EVPLs are determined in separate optimization processes, which can lead to non-optimal solutions for the network and EV owners.

While many studies have explored the impact of CSs on DNs, they have not investigated the impact of increased load resulting from EV charging demand on voltage stability through the mixed-integer linear programming (MILP) model. Some articles have considered voltage stability in other fields, such as charge and discharge management, but do not locate CSs. Reference [19] presents a non-linear model based on a genetic algorithm to determine the appropriate charging and discharging schedule for a CS. Then, suitable locations for CSs are determined to reduce energy loss and improve voltage stability regarding the reactive power of the inverter.

This paper proposes a MILP model for determining the optimal location and size of EVPLs. To obtain more realistic results, we consider a multi-area transportation network and different travel patterns for the movement of EVs between areas on weekdays and weekends. The proposed model aims to maximize the profits of EVPL owners while a linearized index of voltage stability is employed to limit the negative impact of EVPL on DN performance. The difference between this article and previous articles, such as [5], is that the location and capacity of EVPLs and the power exchange with the grid are obtained simultaneously. Therefore, the power exchange of EVPLs is determined, taking into account the DN constraints, and the feasibility of the solution is guaranteed. Also, the difference between the work of this article and references [20, 21] is that the travel pattern of EVs is separated for weekends and working days. Also, a linearized VSI is employed in the proposed model. In general, the innovation of this article is as follows:

• Considering the voltage stability index as a constraint in the planning problem.
• Introducing a linearized formulation for the VSI in the optimization model.
• Regarding different EV travel patterns for working days and weekends.

The rest of this paper is organized as follows. Section 2 introduces the general framework of the problem and its mathematical model. Section 3 is devoted to analyzing the simulation results. Finally, section 4 concludes the results.

2. Model definition

2.1. Model Assumptions

According to Figure (1), we assume that the urban area includes four zones of residential, commercial, industrial, and complex (combination of residential, commercial, and industrial usage) based on the purposes of vehicle travel. We consider three categories for EV daily travels: category 1 includes trips from the residential area to the commercial area, which includes going back and forth from home to shopping centers. Category 2 accounts for trips from the residential area to the industrial area based on the working hours of the industrial centers. Category 3 includes trips from the residential area to the complex area and vice versa. We also assume that there may be travels outside the urban area designated as the external area. In addition, vehicles may also travel from the external area to the internal area [21]. Patterns of the trips can be extracted from historical data.

It is assumed that there is no limit on the number of charging points in the zones. Therefore, all the EVs entering each zone can be placed in the EVPLs. The number of EVs entering/departing each area on working days and weekends and their total battery capacity can be estimated based on statistical studies. The charging tariff is lower than the hourly price of the energy market to encourage EV owners to attend the EVPLs. Also, EV owners who participate in the V2G program are paid for their battery depreciation.

EVs in the residential area can be charged through home chargers, and their charging demand is proportionally allocated to the buses in the residential zone. We assume that EVs leave the residential area with an average charge level of 50%.

2.2. Mathematical model

In this section, a MILP model is proposed to determine the optimal location and capacity of EVPLs. A part of this formulation is inspired by reference [21]. However, the model is extended by incorporating the linearized VSI and different travel patterns.

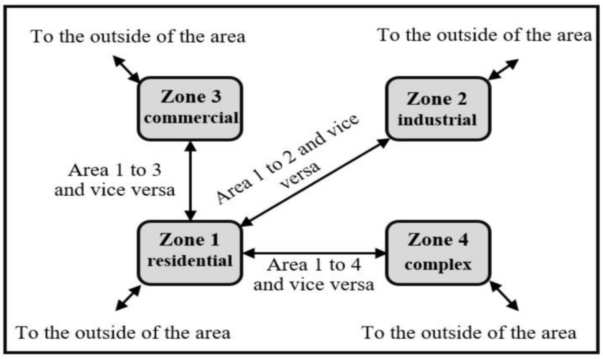

Fig. 1. Types of EV trips between different areas [21].

2.2.1 Objective function

In this paper, the objective function is to maximize the annualized profit of the EVPL owner in the planning horizon. It is the expected income from energy exchange between the EVPLs and network and EVs in different travel patterns minus the total cost of EVPLs. As shown by equation (1), the daily income of EVPLs in the travel pattern p is multiplied by the number of days during a year corresponding to that pattern:

$$Max\ profit = \sum_p \sum_i \sum_t \left(Nd_p.R^{PL}_{p,i,t}\right) - c^{PL} \quad (1)$$

$$R^{PL}_{p,i,t} = R^{EMI}_{p,i,t} + R^{RMI}_{p,i,t} + R^{POI}_{p,i,t} \qquad \forall\, p,i,t \quad (2)$$

$$R^{EMI}_{p,i,t} = \pi^{E}_{p,t}.\left(P^{PL,out}_{p,i,t} - P^{PL,in}_{p,i,t}\right) \qquad \forall\, p,i,t \quad (3)$$

$$R^{RMI}_{p,i,t} = \pi^{R}_{p,t}.re^{PL,out}_{p,i,t} + \pi^{E}_{p,t}.re^{PL}_{p,i,t}\cdot\rho^{del}_{i,t} - \rho^{del}_{i,t}.FOR^{PL}_{i}.\pi^{con}_{p,t}.re^{PL}_{p,i,t} - \rho^{del}_{i,t}.\pi^{V2G}_{p,t}.re^{PL}_{p,i,t}\ ,\ \forall\, p,i,t \quad (4)$$

$$R^{POI}_{p,i,t} = n^{PL}_{p,i,t}.\pi^{tariff}_{t} - \left(P^{PL,out}_{p,i,t} + \rho^{del}_{i,t}.re^{PL}_{p,i,t}\right).C_d + \pi^{G2V}_{p,t}.P^{PL,in}_{p,i,t} - \pi^{V2G}_{p,t}.P^{PL,out}_{p,i,t} \qquad \forall\, p,i,t \quad (5)$$

Equation (2) shows the three components of the EVPLs' income: 1. The income from the energy exchange of EVPLs with the network; 2. The earnings from the reserve sale and its deployment during contingencies; 3. The revenue from the energy exchange with EVs. These components are calculated with equations (3)-(5) respectively. Equation (4) states that if the EVPL fails to deliver the allocated reserve after being called during a contingency, it will face a penalty based on the hourly energy price. According to equation (5), EVPL receives a parking charge from the EV owner based on a fixed tariff and a charging fee for charging their EV battery. However, there are payments to the EV owners for the power purchase from their EV battery in the V2G mode, plus the battery depreciation cost. The second term of equation (1) represents the total cost for EVPLs and is calculated by equation (6). It includes the installation cost and the cost of increasing network loss. To avoid the excessive increase of network power loss, we assume that the EVPL owner is responsible for the incremental cost of network losses compared to the initial state of the network.

$$c^{PL} = \sum_i c_i^{ins} + c^{loss} \quad (6)$$

$$c_i^{ins} = \sum_{b|b\in zone(i)}(c^{fix} \cdot si_b^{PLA} \cdot CRF) + c_i^{var} \ , \ \forall i \quad (7)$$

$$c_i^{var} = \sum_{b|b\in zone(i)}([A \cdot c^{land} + c^{eq}] \cdot CRF + c^{M}).ns_b^{PLA} \quad \forall i \quad (8)$$

$$si_b^{PLA} \cdot ns_b^{PLA,min} \le n_b^{PLA} \le si_b^{PLA}.ns_b^{PLA,max} \quad \forall b \quad (9)$$

$$min_i^{PL} \le \sum_b si_{b,i}^{PLA} \le max_i^{PL} \quad i = 2,3,4\ , \ \forall b \quad (10)$$

$$si_b^{PLA} \le Can_b^{PLA} \quad \forall b$$

(11)

$$\sum_{k\in(i\neq 1)} u_{p,b,k,t}^{PL} \le ns_b^{PLA} \quad \forall p,b,k,t \quad (12)$$

$$NS_i^{PL} = \sum_b ns_{b,i}^{PLA} \quad \forall b,i \quad (13)$$

According to equation (7), installing an EVPL requires fixed and variable costs. The fixed cost is related to the costs of obtaining the construction permit and the municipal costs of installing the EVPL. The fixed cost is multiplied by a binary variable corresponding to each bus, indicating the selection of the bus for the construction of the EVPL. In equation (8), the variable cost includes the purchasing cost of the land required for the EVPL, the cost of charging equipment, and their maintenance cost, all of which are proportional to the number of charging points in the EVPL. According to equations (9) and (10), the number of charging points in the EVPL of each bus and the number of EVPL installed in each area are limited between the minimum and maximum values. Equation (11) shows the candidacy of a bus for EVPL installation. According to (12), at any time, the total number of charging points occupied in each EVPL cannot exceed the number of installed chargers in that EVPL. Equation (13) calculates the number of installed chargers in each area. In equations (7) and (8), CRF is the capital recovery factor that converts investment cost into annualized cost. Equation (14) calculates the CRF.

$$CRF = \frac{d}{(1-(1+d)^{-n})} \quad (14)$$

Equation (15) calculates the network losses. As shown by equation (16), the incremental cost of network losses due to EVPLs is equal to the difference between hourly losses with and without the presence of EVPLs multiplied by the hourly energy price in the desired travel pattern. Linearized AC load flow equations are employed in the radial DN to calculate the square of the currents for determining network loss [21].

$$loss_{p,t} = \sum_l R_l \left(i_{p,l,t}\right)^2 \quad \forall p,t \quad (15)$$

$$c^{loss} = Nd_p.\sum_p \sum_t (loss_{p,t} - loss_{p,0,t}) \cdot \pi_{p,t}^{E} \quad (16)$$

2.2.2 Mathematical model of the presence of EVs in the EVPLs

Figures 2 and 3 show the arrival/departure of EVs to/from each zone and the power exchange of EVs with the network.

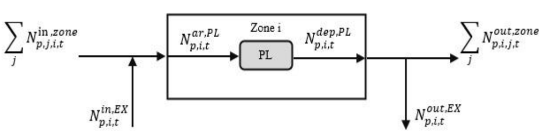

Fig. 2. Diagram of entering and departure of EVs in EVPLs of area i [21].

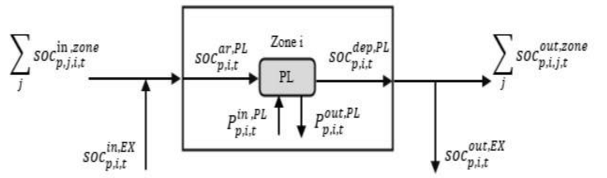

Fig. 3. Diagram of EV charging/discharging in the EVPLs of area i [21].

According to equations (17) and (18), the total number of EVs arriving/departing at/from the EVPLs of each area is equal to the total number of EVs entering/leaving from/to other zones and the external area. Equation (19) calculates the number of EVs in area *i* based on the number of EVs in the previous period and the number of incoming and outgoing ones. As assumed before, all EVs entering each area can be placed in the EVPLs of that area. Therefore, equation (20) states that the total number of EVs parked in the parking charging stations in each bus is equal to the total number of occupied chargers in that area.

$$n_{p,i,t}^{ar,PL} = N_{p,i,t}^{in,EX} + \sum_j N_{p,j,i,t}^{in,zone} \quad \forall p,i,t \quad (17)$$

$$n_{p,i,t}^{dep,PL} = N_{p,i,t}^{out,EX} + \sum_j N_{p,i,j,t}^{out,zone} \quad \forall p,i,t \quad (18)$$

$$n_{p,i,t}^{PL} = N_{i,t0}^{PL}\Big|_{t=1} + n_{p,i,t-1}^{PL}\Big|_{t>1} + n_{p,i,t}^{ar,PL} - n_{p,i,t}^{dep,PL} \quad (19)$$

$$\sum_{b\epsilon i}\sum_k u_{p,b,k,t}^{PL} = n_{p,i,t}^{PL} \quad i = 2,3,4 \ ,\forall p,b,k,t \quad (20)$$

Equations (21) and (22) determine the total state of charge (SoC) of the EVs that arrived/departed at/from EVPLs of zone i. Equation (23) calculates the average travel time from zone i to j according to their distance and the average speed of EVs. Equation (24) states that an EV that leaves zone i to j loses some of its SoC when entering the destination zone [21]. According to equations (25) and (26), the total SoC of the EVs that enter or exit the area cannot be greater than their battery capacity.

$$soc_{p,i,t}^{ar,PL} = SOC_{p,i,t}^{in,Ex} + \sum_j soc_{j,i,t}^{in,zone} \quad \forall p,i,t \quad (21)$$

$$soc_{p,i,t}^{dep,PL} = SOC_{p,i,t}^{out,Ex} + \sum_j soc_{i,j,t}^{out,zone} \quad \forall p,i,t \quad (22)$$

$$\alpha_{i,j} = \frac{L_{i,j}}{S_{i,j}} \quad \forall i,j \quad (23)$$

$$soc_{p,i,j,t+\alpha_{i,j}}^{in,zone} = soc_{pj,i,t}^{out,zone} - N_{i,j,t}^{out,zone} \cdot L_{i,j} \cdot P_{i,j}^{Fuel} \quad \forall p,i,j,t \quad (24)$$

$$soc_{p,i,j,t}^{in,zone} \le C_{p,i,j,t}^{in,zone} \quad \forall p,i,j,t \quad (25)$$

$$soc_{p,i,j,t}^{out,zone} \le C_{p,i,j,t}^{out,zone} \quad \forall p,i,j,t \quad (26)$$

On the other hand, the power exchange between the EVPLs and the network changes the SoC of EVs. According to (27), the SoC of the EVPLs of each area at each period depends on the SoC of the remaining EVs at the previous hour, the charging and discharging power regarding the charge and discharge efficiencies, and the SoC of the EVs arriving/departing the EVPLs. In equation (28), we assumed that the SoC of the EVs leaving the EVPLs is proportional to the ratio of the EVs leaving each area to the total EVs in that area. In equations (29) and (30), the SoC of the EVs of each travel is considered proportional to the number of EVs in that travel. Equation (31) limits the SoC of EVPL to the minimum and maximum percentage of its total battery capacity.

$$soc_{p,i,t}^{PL} = soc_{i,0}^{PL}\big|_{t=1} + soc_{p,i,t-1}^{PL}\big|_{t>1} + P_{p,i,t}^{PL,in} \cdot \eta_c - \frac{P_{p,i,t}^{PL,out}}{\eta_d} + soc_{p,i,t}^{ar,PL} - soc_{p,i,t}^{dep,PL} \quad ,\forall p,i,t \tag{27}$$

$$soc_{p,i,t}^{dep,PL} = soc_{p,i,t}^{PL} \cdot \frac{\left(N_{p,i,t}^{out,Ex} + \sum_j N_{p,i,j,t}^{out,zone}\right)}{n_{p,i,t}^{PL}} \quad , \forall p,i,t \tag{28}$$

$$soc_{p,i,j,t}^{out,Zone} = soc_{p,i,t}^{dep,PL} \cdot \frac{N_{p,i,j,t}^{out,Zone}}{n_{p,i,t}^{dep,PL}} \quad , \forall p,i,j,t \tag{29}$$

$$soc_{p,i,t}^{out,Ex} = soc_{p,i,t}^{dep,PL} \cdot N_{p,i,t}^{out,Ex} / n_{p,i,t}^{dep,PL} \quad \forall p,i,t \tag{30}$$

$$soc_i^{EV,min} . c_{p,i,t}^{PL} \leq soc_{p,i}^{PL} \leq soc_i^{EV,max} . c_{p,i,t}^{PL} \quad \forall p,i \tag{31}$$

According to equation (32), the charging power of the EVPL is limited by the number of EVs in the EVPL and their charging rate. In equation (33), the maximum output power of the EVPL is constrained by the number of EVs, their discharge rate and a percentage of the EV's SoC that is determined in advance through a contract between the EV owners and the EVPL [21].

$$P_{p,i,t}^{PL,in} \leq \Gamma_i^{PL} \cdot n_{p,i,t}^{PL} \qquad i = 2,3,4\ ,\ \forall p,t \tag{32}$$

$$P_{p,i,t}^{PL,out} \leq min\{\Gamma_i^{PL} \cdot n_{p,i,t}^{PL}; soc_{p,i,t}^{PL} \cdot \phi_i^{PL}\} \qquad i = 2,3,4\ ,\forall p,t \tag{33}$$

$$P_{p,i,t}^{PL,out} + r_{p,i,t}^{PL,out} \leq min\{\Gamma_i^{PL} \cdot n_{p,i,t}^{PL}, soc_{p,i,t}^{PL} \cdot \text{K}_i^{PL}\} \qquad i = 2,3,4 \quad ,\forall p,t \tag{34}$$

Equation (34) states that the total output power and the scheduled reserve cannot be greater than the discharging power of the EVPL and a charge level that can be used in the charging points.

2.2.3 Modeling the effect of EVPLs on the distribution network

Here, the contribution of the power exchanges of EVPLs to the demand of load points in the DN is formulated. Equation (35) indicates that the input power to zone *i* for charging EVs is equal to the sum of the input power to EVPLs installed at buses in that area. Equation (36) expresses the same concept for the output power of zone i. Based on equations (37) and (38), the input/output power to/from the EVPL installed at bus b is less than or equal to the number of occupied charging points in that EVPL multiplied by their maximum charging/discharge rate. In the first zone (residential zone), the V2G feature is not assumed for home chargers. Therefore, the discharging power for this zone is zero, as stated by equation (39).

$$p_{p,i,t}^{PL,in} = \sum_{b|b\in zone(i)} p_{p,b,t}^{PLA,in} \qquad i = 2,3,4\ ,\forall p,t \tag{35}$$

$$p_{p,i,t}^{PL,out} = \sum_{b|b\in zone(i)} p_{p,b,t}^{PLA,out} \qquad i = 2,3,4\ ,\forall p,t \tag{36}$$

$$p_{p,b,t}^{PLA,in} \leq \Gamma_i^{PL} \cdot \sum_k u_{p,b,k,t}^{PL} \qquad \forall p,b,t \tag{37}$$

$$p_{p,b,t}^{PLA,out} \leq \Gamma_i^{PL} \cdot \sum_k u_{p,b,k,t}^{PL} \qquad \forall p,b,t \tag{38}$$

$$P_{p,i,t}^{PL,out}\big|_{i=1} = 0 \qquad i = 1\ ,\ \forall p,t \tag{39}$$

AC load flow equations are employed to calculate line currents, network losses and bus voltages. Equations (40) and (41) show the nodal active and reactive power balance equations. Equation (42) indicates the relation of the voltage of consecutive buses, which depends on the active and reactive power flows on the line between them and the line parameters. Equation (43) calculates the line current according to the active and reactive power flows and the voltage of its receiving end. Equations (42) and (43) are nonlinear. However, they can be linearized through the piecewise linear approximation method [5]. Equations (44) and (45) limit the bus voltages and line currents to their acceptable range.

$$p_{p,b,t}^{Sys,in}\big|_{b=1} + p_{p,b,t}^{PLA,out} - p_{p,b,t}^{PLA,in} - \sum_l \left(p_{p,l,t}^{line} + R_l\left(i_{l,t}\right)^2\right) = p_{p,b,t}^{D} \qquad \forall p,b,t \tag{40}$$

$$q_{p,b,t}^{Sys,in} - \sum_l \left(q_{p,l,t}^{line} + X_l\left(i_{p,l,t}\right)^2\right) = q_{p,b,t}^{D} \quad \forall p,b,t \tag{41}$$

$$v_{p,b,t}{}^2 - 2\left(R_l . p_{p,l,t}^{Line} + X_l . q_{p,l,t}^{Line}\right) - Z_l^2 \cdot i_{p,l,t}^2 = v_{p,b',t}{}^2 \qquad \forall p,b,t \tag{42}$$

$$\left(i_{p,l,t}\right)^2 = \frac{\left(p_{p,l,t}^{Line}\right)^2 + \left(q_{p,l,t}^{Line}\right)^2}{\left(v_{p,b',t}\right)^2} \qquad \forall p,l,t \tag{43}$$

$$v_b^{min} \leq v_{p,b,t} \leq v_b^{max} \qquad \forall p,b,t \tag{44}$$

$$-I_L^{max} \leq I_{p,l,t} \leq I_L^{max} \qquad \forall p,b,t \tag{45}$$

An important issue in the allocation of EVPLs is maintaining the voltage stability of the DN. To address this concern, the VSI of the DN is formulated and incorporated into the constraints of the optimization problem. According to Fig. 4, the VSI for each bus without and with the presence of EVPL power exchanges are determined by equations (46) and (47).

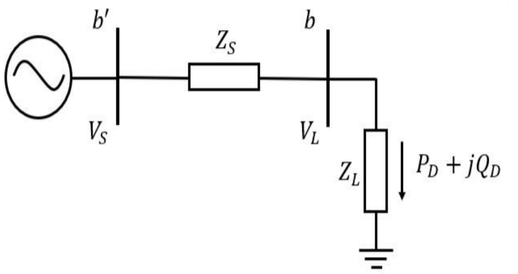

Fig. 4. Equivalent circuit of the distribution system for calculation of VSI [18].

$$VSI_{b,t} = \left|V_{b',t}\right|^4 - 4.\left[p_{b,t}^D \cdot R_l + q_{b,t}^D \cdot X_l\right].\left|V_{b',t}\right|^2 - 4.\left[\left(p_{b,t}^D\right) \cdot X_l - q_{b,t}^D \cdot R_l\right]^2 \qquad \forall b,t \tag{46}$$

$$VSI_{p,b,t} = \left|V_{p,b',t}\right|^4 - 4.\left[\left(p_{p,b,t}^D + p_{p,b,t}^{PLA,in} - p_{p,b,t}^{PLA,out}\right) \cdot R_l + q_{p,b,t}^D \cdot X_l\right].\left|V_{p,b',t}\right|^2 - 4.\left[\left(p_{p,b,t}^D + p_{p,b,t}^{PLA,in} - p_{p,b,t}^{PLA,out}\right) \cdot X_l - q_{p,b,t}^D \cdot R_l\right]^2 \qquad \forall p,b,t \tag{47}$$

Equation (47) is nonlinear. However, linearized expressions for the quadratic terms are available through the piecewise linear approximation method [5]. The same approach can be applied to the quadratic term for linearizing the fourth-order term of the voltage. References [22] and [23] introduced an approximative linearization of the product of two non-binary variables, as presented in the Appendix. We employed this approach for linearizing the product of the terms $\left(p_{p,b,t}^{PLA,in} - p_{p,b,t}^{PLA,out}\right)$ and $\left|V_{p,b',t}\right|^2$ through equations (48)-(53). Therefore, the proposed optimization problem remains an MILP problem.

$$y_{p,b,t} = \left(p_{p,b,t}^{PLA,in} - p_{p,b,t}^{PLA,out}\right). R_l \tag{48}$$

$$|V_{min}|^2 \leq \left|V_{p,b',t}\right|^2 \leq |V_{max}|^2 \qquad \forall p,b,t \tag{49}$$

$$-ns_b^{PLA,max} . \Gamma_b^{PL} . R_l \leq y_{p,b,t} R_l \leq ns_b^{PLA,max} . \Gamma_b^{PL} . R_l \qquad \forall p,b,t \tag{50}$$

$$Z_{p,b,t} = \left[\left(p_{p,b,t}^{PLA,in} - p_{p,b,t}^{PLA,out}\right) \cdot R_l\right].\left|V_{p,b\prime,t}\right|^2 \qquad \forall\, p,b,t \qquad (51)$$

$$V_{min}{}^2 . y_{p,b,t} \le Z_{p,b,t} \le V_{max}{}^2 . y_{p,b,t} \qquad \forall\, p,b,t \qquad (52)$$

$$y_{min}(b). V_{p,b\prime,t}{}^2 \le Z_{p,b,t} \le y_{max}(b). V_{p,b',t}{}^2 \quad \forall\, p,b,t \quad (53)$$

3. Simulation results

We implemented the proposed model for placing and scheduling EVPLs in the 37-bus IEEE test system. Figure 5 shows the single-line diagram of this system and the border of traffic zones[21]. The voltage level of the network is 4.8 kV, and the peak active and reactive demand of this network is 2.5 MW and 1.5 MVAR, respectively. The line impedances are risen by factor 2 to increase the stress on the DN. We assume that the EVs in the residential area use 3 kW chargers, while in the EVPLs of other zones, the 11 kW chargers are used [25,26]. Figures 6 and 7 show the hourly load profile of the system and the hourly energy and reserve prices. Table I presents other parameters required for implementing the model. The arrival of EVs at the zones and their departure are extracted from reference [20]. We assume that the number of EVs in each travel is proportional to the total battery capacity of the EVs in that travel, as illustrated by Fig. 8.

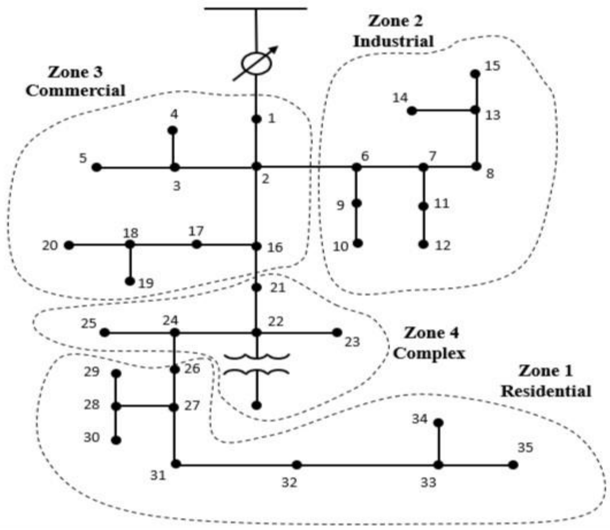

Fig. 5. Single line diagram of the IEEE 37-bus test system [21].

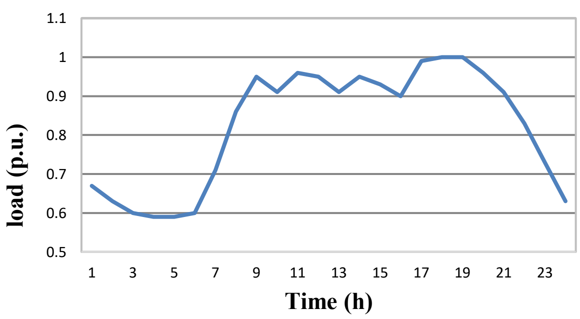

Fig. 6. Hourly load profile [26]

It is assumed that energy prices in V2G and G2V modes are 0.02 \$/kWh lower than the market price, while the energy price in contingency conditions (π_t^{con}) is 20% higher than the market price.

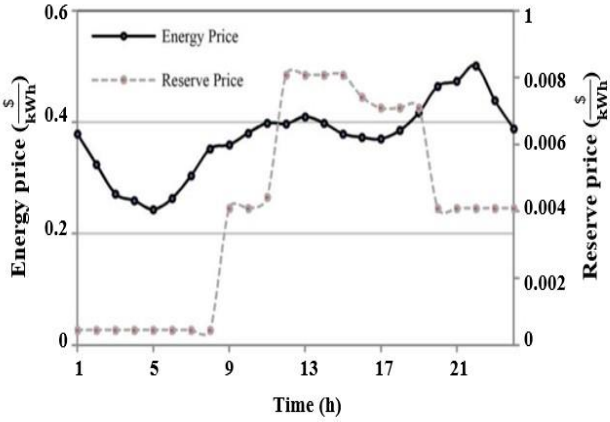

Fig. 7. Hourly market prices of energy and reserve [5].

Table I. Value of parameters in the simulation [20].

Parameter	value	Parameter	value
η_d , η_c	0.9	C^{land}	407 (\$/m^2)
FOR	0.02	c^{fix}	18000 (\$)
$soc_i^{EV,min}$	0.15	C^M	30 (\$/year)
$soc_i^{EV,max}$	0.90	C^{eq}	2000 (\$/year)
$v_b{}^{min}, v_b{}^{max}$	0.9,1.1 (P.U)	A	25 (m^2)
ϕ_i^{PL}	0.30	$P_{i,j}^{Fuel}$	0.2 (kWh/km)
κ_i^{PL}	0.70	d	0.10
C_d	0.075 ($\$/kWh$)	n	5 (year)

Table (II) shows the information used to model different patterns of EV travel on weekdays and weekends [27]. At weekends, work trips, Recreation and shopping trips and all trips have decreased by about 35, 60, and 50 percent, respectively. Since most vehicle owners make more than one type of trip during the day, the total participation of all trips is more than 100%. The optimization problem is solved in the GAMS environment using the CPLEX solver.

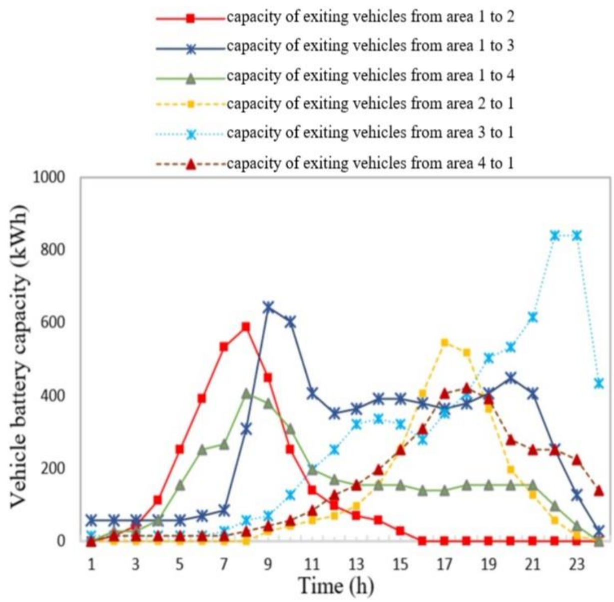

Fig. 8. Battery capacity of EVs entering and exiting different zones [20].

Table II. The share of activities in people's travel during weekdays and weekends [27].

Activity Purpose	Participation in weekdays (%)	Participation in weekends (%)
Work	82.6	28.6
Shopping	59.8	33.0
Personal business	26.3	13.2
Recreation	69.9	44.0

The results of the allocation of EVPLs have been compared in the following cases:
Case 1: without voltage stability constraint and with the same travel pattern for all weekdays.
Case 2: with voltage stability constraint and the same travel patterns for all weekdays.
Case 3: without voltage stability constraint and with different travel patterns for weekdays
Case 4: with voltage stability constraint and different travel patterns for weekdays

Table III compares the location and capacity of the EVPLs in the case studies. The VSI of each bus in the four cases is shown in Fig. 9. According to Fig. 9, in case 1, The VSI decreases with the presence of parking lots compared to the case where parking lots are not present. However, the VSI has improved in almost all buses when a minimum value is defined as a constraint in case 2. The minimum value of VSI in this case is 0.736. In case 3, the VSI on working days is similar to case 2 because the EV travel pattern on weekdays is the same. Also, the VSI in case 4 and on weekends has improved due to the reduction in EV trips and the consequent decrease of their charging demand in the EVPLs. Due to the constraint for the minimum value of VSI, most of the buses in Case 4 have a higher VSI compared with Case 1. Generally speaking, the VSI can be improved to a great extent by the appropriate location of EVPLs and their power exchanges with the network. Table IV compares the revenue and cost components for the EVPLs in the four cases.
According to Table IV, a significant part of EVPL revenues is from the interaction with EV owners. However, the profit of the EVPL owner reduces when the VSI constraint and different EV travel patterns are introduced. The reason is that the EVPLs have to be constructed in less economical buses to maintain VSI above its lower bound. Also, fewer EVs will be present in the EVPLs on weekends, which reduces the revenue of EVPLs. An affecting factor in the proposed model is the minimum requirement of the VSI for each bus. Fig. 10 shows the effect of changing the minimum VSI constraint from 0.74 to 0.77 on the VSI of buses. Accordingly, the optimal location and size of EVPLs are demonstrated in Table V.

Table III. location and capacity of EVPLs in different cases

Case	Location and capacity of EVPLs in the buses	Total capacity of EVPLs in the zones
Case 1	B6(91) – B7(95) B2(92) – B3(72) B21(100) – B22(69)	I2(186) I3(164) I4(169)
Case 2	B6(93) – B7(93) B2(92) – B3(72) B21(100) – B25(69)	I2(186) I3(164) I4(169)
Case 3	B6(88) – B7(98) B2(89) – B3(75) B21(84) – B22(85)	I2(186) I3(164) I4(169)
Case 4	B6(95) – B9(91) B2(99) – B3(65) B21(99) – B22(70)	I2(186) I3(164) I4(169)

Table IV. Revenue and cost components (M$) in different cases.

	Case 1	Case 2	Case 3	Case 4
R(EMI)	-1.2519	-1.1909	-1.1068	-1.0612
R(RMI)	0.8074	0.7728	0.7182	0.6922
R(POI)	5.9713	5.9128	5.3077	5.2636
Total Revenue	5.5268	5.4946	4.9191	4.8946
Investment costs	1.7110	1.7110	1.7110	1.7110
C(loss)	0.0575	0.0542	0.0536	0.0513
C(PL)	1.7685	1.7651	1.7646	1.7622
Profit	3.7583	3.7295	3.1545	3.1324

Table V. Capacity of EVPLs in the buses and zone for different values of minimum VSI.

Min VSI constraint	Capacity of EVPLs at the buses	Capacity of EVPLs in each zones
without constraint (Case 3)	B6(88), B7(98) B2(89), B3(75) B21(84), B22(85)	I2(186) I3(164) I4(169)
0.74	B6(99), B7(87) B2(88), B3(76) B21(84), B22(85)	I2(186) I3(164) I4(169)
0.75	B6(99), B7(87) B2(97), B3(67) B21(98), B22(71)	I2(186) I3(164) I4(169)
0.76 (Case 4)	B6(95), B9(91) B2(99), B3(65) B21(99), B22(70)	I2(186) I3(164) I4(169)
0.77	B6(94), B8(92) B2(93), B3(71) B21(100), 24(69)	I2(186) I3(164) I4(169)

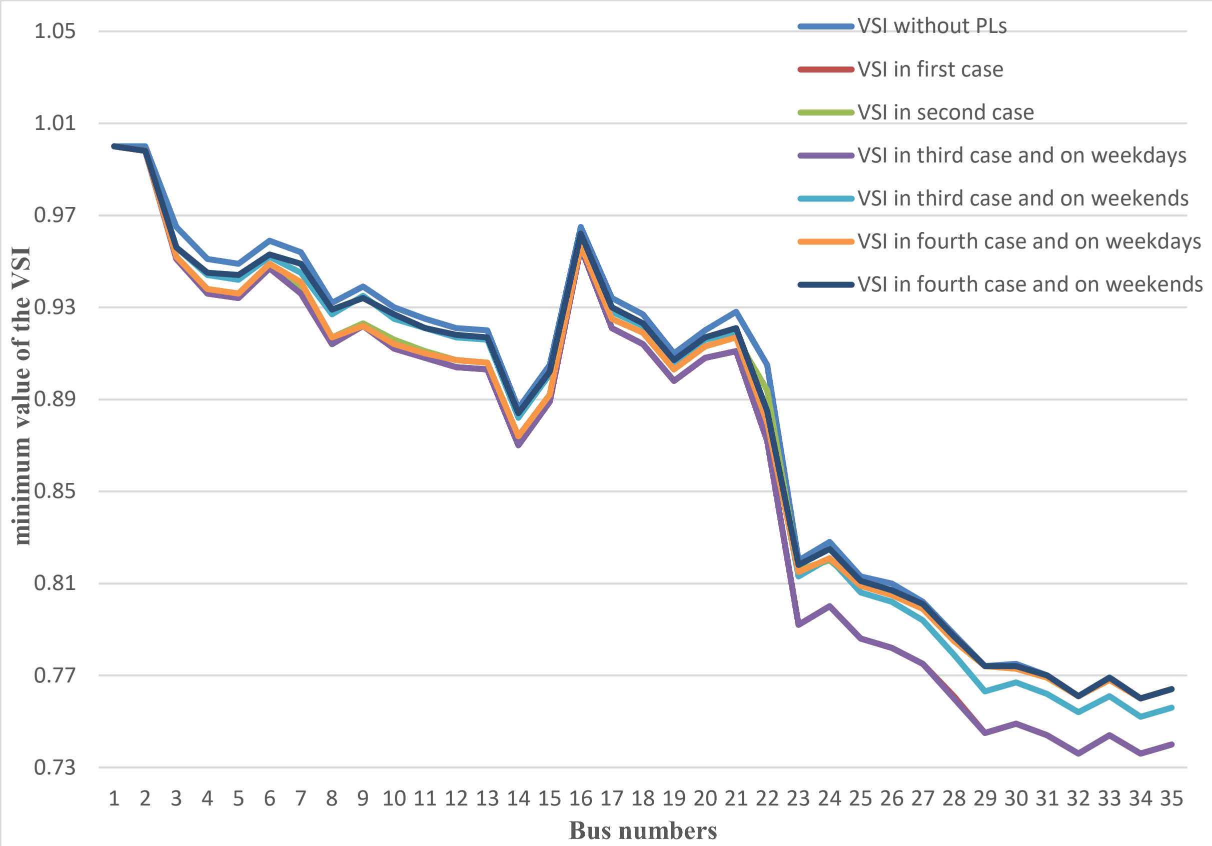

Fig.9. Minimum value of VSI in each bus in the four cases.

According to Fig. 10, by increasing the minimum VSI constraint, it is significantly improved in the end buses. Therefore, applying the appropriate VSI constraint can prevent voltage instability in the DN. With a higher value of the minimum VSI requirement, the power exchange from EVs to the grid increases. Therefore, the cost of incremental losses and the income from the interaction between EVPLs and the energy market has increased. However, the profit of the EVPL owner reduces because the locations of EVPLs change to places that are less economical to respond to the restriction caused by the VSI constraint.

4. Conclusion

The proposed model can allocate EVPLs by considering the constraints of the DN and the effects of the transportation network on the DN. It also takes into account the VSI and different travel patterns on weekdays. The results show that with a constraint on the minimum VSI value, the EVPL owner may have to build EVPLs in less economical places to improve the VSI. Therefore, the profit of the investor decreases. Also, by considering a different pattern for the travel of EVs on weekdays and weekends, the EVPL profit reduces due to the decrease in the presence of EVs in EVPLs on weekends.

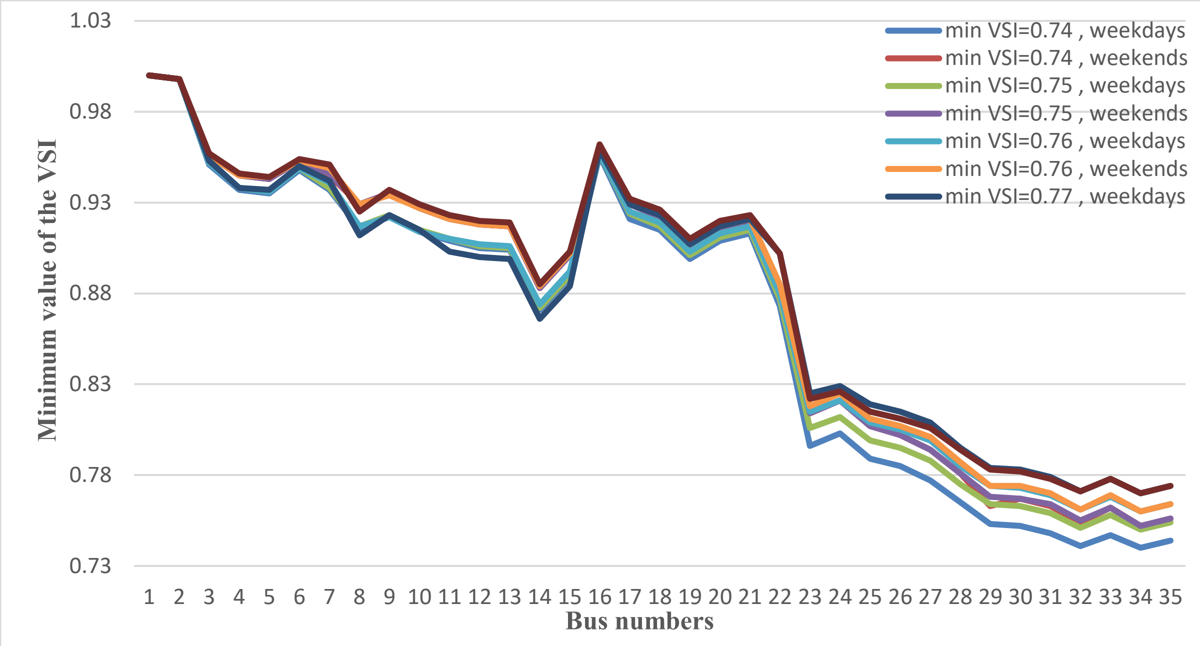

Fig. 10. VSI for different minimum constraints.

5. Nomenclature

indices

b	Index of buses
i, j	Index of zones
k	Index of charging points
l	Index of lines
p	Index of travel patterns on weekdays and weekends
t	Index of time

parameters

A	Required land to install each charging point (m^2)
C_d	Depreciation cost of EV battery due to V2G (\$)
$c_{p,i,t}^{PL}$	The battery capacity of EVs in pattern p in EVPLs of area i at time t
c^M, c^{eq}	Cost of purchase and maintenance of charger (\$)
c^{land}	Cost of land for installing charging point $(\$/m^2)$
c^{fix}	Fixed cost for the construction of an EVPL (\$)
$C_{p,i,j,t}^{in,zone}$	Battery capacity of EVs that enter from zone i to j in pattern p at time t (kWh)
$C_{p,i,j,t}^{out,zone}$	Battery capacity of EVs that exit from zone i to j in pattern p at time t (kWh)
Can_b^{PLA}	Binary parameter which is 1 if bus b is candidate for the construction of EVPL.
FOR_i^{PL}	Failure probability of EVPL in the area i to deliver the reserve when being called.
$I_L{}^{max}$	Maximum allowed current of line L (P.U)
$L_{i,j}$	Distance between area i and j (km)
min_i^{PL}	Minimum number of installed EVPLs in area i
max_i^{PL}	Minimum number of installed EVPLs in area i
$N_{p,i,t}^{in,EX}$	Number of EVs arriving from the external area to area i at time t in pattern p
$N_{p,i,t}^{out,EX}$	Number of outgoing EVs from area i to the external area at time t in pattern p
$N_{p,j,i,t}^{in,zone}$	Number of EVs that enter from zone i to j in pattern p at time t
$N_{p,i,j,t}^{out,zone}$	Number of EVs that exit from zone i to j in pattern p at time t
$ns_b^{PLA,min}$	Minimum number of chargers that can be installed in the EVPL of bus b
$ns_b^{PLA,max}$	Maximum number of chargers that can be installed in the EVPL of bus b
$P_{i,j}^{Fuel}$	Average energy consumption of EV in traveling from area i to j (kWh/km)
$p_{p,b,t}^D$	Active power demand of bus b in pattern p at time t (kW)
$q_{p,b,t}^D$	Reactive power demand of bus b in pattern p at time t (kW)
$R_{b,b'}$	Resistance of the line between buses b and b′ (p.u.)
$X_{b,b'}$	Reactance of the line between buses b and b′ (p.u.)
$Z_{b,b'}$	Impedance of the line between buses b and b′ (p.u.)
$S_{i,j}$	Average speed of EV in travel from area i to j (km/h)
$soc_i^{EV,min}$	Minimum SoC of EV battery in area i (%)
$soc_i^{EV,max}$	Minimum SoC of EV battery in area i (%)
$v_b{}^{max}$	Upper bound of voltage of bus b (p.u.)
$v_b{}^{min}$	Lower bound of voltage of bus b (p.u.)
$VSI_{p,t}^b$	Voltage stability index of bus b in pattern p at time t
η_d, η_c	Charging and discharging efficiency of chargers(%)
ϕ_i^{PL}	Minimum SoC required by the EV when leaving area i (kWh)
Γ_i^{PL}	Charging and discharging rates of the chargers in area i (kW)
κ_i^{PL}	Utilizable SoC of EVs in EVPL i according to the contract with EV owners (%)
$\pi_{p,t}^E, \pi_{p,t}^R$	Market price of energy and reserve power in pattern p at time t (\$/kWh)
$\pi_{p,t}^{con}$	Energy price during contingencies in pattern p at time t (\$/kWh)
π^{Tariff}	Parking fee $(\$/h)$

$\pi_{p,t}^{V2G}, \pi_{p,t}^{G2V}$ Price of buying/selling energy from/to EVs in pattern *p* at time *t* ($/kWh)
$I\rho_{i,t}^{del}$ Probability of EVPL *i* for being called to deliver reserve power at time *t*

Variables

C^{PL} Total cost of EVPLs ($)
c_i^{ins} Installation cost of EVPLs in area *i* ($)
c_t^{loss} Cost of energy loss in the network at time *t* ($)
c_i^{var} Variable construction cost of EVPLs in area *i* ($)
$loss_{p,t}$ Network Losses in pattern *p* at time *t* in presence of EVPLs (kW)
$loss_{p,0,t}$ Initial network losses in pattern *p* at time *t* without the presence of EVPLs (kW)
$n_{p,i,t}^{ar,PL}$ Number of EVs arriving at EVPLs of zone *i* in pattern *p* at time t
$n_{p,i,t}^{dep,PL}$ Number of EVs departing the EVPLs of zone *i* in pattern *p* at time t
$n_{p,i,t}^{PL}$ Number of EVs in the EVPLs of zone *i* in pattern *p* at time t
ns_b^{PLA} Number of chargers in the EVPL of bus *b*
NS_i^{PL} Total number of chargers in the EVPLs of area *i*
$profit^{PL}$ Profit of EVPL owner ($)
$p_{p,b,t}^{Sys,in}$ active power input from the upstream network to bus *b* in pattern *p* at time *t* (kW)
$p_{p,b,b',t}^{line}$ Active power flow on the line between buses *b* and *b*' in pattern *p* at time *t* (kW)
$P_{p,i,t}^{PL,in}$ Input power to EVPLs of zone *i* in pattern *p* at time *t* (kW)
$P_{p,i,t}^{PL,out}$ Output power from EVPLs of zone *i* in pattern *p* at time *t* (kW)
$P_{p,b,t}^{PLA.in}$ Input power to EVPL on bus *b* in pattern *p* at time *t* (kW)
$P_{p,b,t}^{PLA,out}$ output power of EVPL on bus *b* in pattern *p* at time *t* (kW)
$q_{p,b,t}^{Sys,in}$ reactive power input from the upstream network to bus *b* in pattern *p* at time *t* (kW)
$q_{p,b,b',t}^{line}$ Reactive power flow on the line between buses *b* and *b*' in pattern *p* at time *t* (kW)
$re_{p,i,t}^{PL,out}$ Reserve power of EVPL *i* that is being called in pattern *p* at time *t* (kW)
$soc_{p,i,t}^{PL}$ SoC of EVs in EVPLs of zone *i* in pattern *p* at time *t* (kWh)
$soc_{p,i,t}^{ar,PL}$ SoC of EVs arrived in EVPLs of zone *i* in pattern *p* at time *t* (kWh)
$soc_{p,i,t}^{dep,PL}$ SoC of EVs departed EVPLs of zone *i* in pattern *p* at time *t* (kWh)
$soc_{p,i,t}^{in,Ex}$ SoC of EVs arrived in zone *i* from the external zone in pattern *p* at time *t* (kWh)
$soc_{p,i,t}^{out,Ex}$ SoC of EVs leaving zone *i* to the external zone in pattern *p* at time *t* (kWh)
$soc_{p,i,j,t}^{in,zone}$ SoC of EVs arrived in zone *i* from zone *j* in pattern *p* at time *t* (kWh)
$soc_{p,i,j,t}^{out,zone}$ SoC of EVs leaving zone *i* to *j* in pattern *p* at time *t* (kWh)
$soc_{p,i,t}^{dep,zone}$ SoC of EVs leaving zone *i* in pattern *p* at time *t* (kWh)
$V_{p,b,t}$ Voltage of bus *b* in pattern *p* at time *t* (p.u.)
$i_{p,l,t}$ Current of line *l* in pattern *p* at time *t* (p.u.)
si_b^{PLA} Binary variable which is 1 if bus *b* is selected to construct an EVPL
$u_{p,b,k,t}^{PL}$ Binary variable which is 1 if charging point *k* on bus *b* is occupied by an EV in pattern *p* at time

6. References

[1] M. Mozaffari, H. Askarian Abyaneh, M. Jooshaki and M. Moeini-Aghtaie, "Joint Expansion Planning Studies of EV Parking Lots Placement and Distribution Network," *IEEE Transactions on Industrial Informatics*, vol. 16, no. 10, pp. 6455-6465, Oct. 2020.

[۲] مهدی تورانی، محمدرضا آقاابراهیمی، حمیدرضا نجفی، «برنامه‌ریزی محدوده پارکینگ خودروهای الکتریکی و شارژ و دشارژ آن به‌منظور بهبود قابلیت‌اطمینان در شبکه‌های هوشمند»، مجله مهندسی برق دانشگاه تبریز، جلد ٤٧، شماره ٢، صفحات ٤١٣-٤٢٢، ١٣٩٦.

[۳] رامین شریک آبادی، امیر عبداللهی، مسعود رشیدی نژاد، مهدی شفیعی، «مدل‌سازی امکانی- احتمالاتی پارکینگ خودروهای برقی با رویکرد بهبود انعطاف‌پذیری در برنامه‌ریزی مشارکت امنیت مقید واحدهای نیروگاهی»، مجله مهندسی برق دانشگاه تبریز، جلد ٥٣، شماره ٣، صفحات ٢٠٩-٢٢٢، ١٤٠٢.

[۴] محمدرضا فلاح‌زاده، علی زنگنه، «مدل برنامه‌ریزی دوسطحی پیشنهاددهی قیمت بهینه تجمیع‌کننده خودروهای الکتریکی در شبکه توزیع»، مجله مهندسی برق دانشگاه تبریز، جلد ۴۸، شماره ۴، صفحات ۱۶۹۹-۱۷۰۹، ۱۳۹۷.

[5] N. Neyestani, M. Y. Damavandi, M. Shafie-Khah, J. Contreras and J. P. S. Catalão, "Allocation of Plug-In Vehicles' Parking Lots in Distribution Systems Considering Network-Constrained Objectives," *IEEE Transactions on Power Systems*, vol. 30, no. 5, pp. 2643-2656, Sep. 2015.

[6] M. H. Amini, M. P. Moghaddam and O. Karabasoglu, "Simultaneous allocation of electric vehicles' parking lots and distributed renewable resources in smart power distribution networks," *Sustainable Cities and Society*, vol. 28, pp. 322-342, Jan. 2017.

[۷] پویا حسین زاده، خلیل گرگانی فیروزجاه، عبدالرضا شیخ الاسلامی، «برنامه‌ریزی ایستگاه شارژ خودروی الکتریکی با هدف بیشینه‌سازی سود خودروها و اپراتور ایستگاه در شرایط عدم قطعیت»، مجله مهندسی و مدیریت انرژی دانشگاه کاشان، دوره ۹، شماره ۳، صفحات ۲-۱۳، ۱۳۹۸.

[8] M. Moradijoz, F. Moazzen, S. Allahmoradi, M. P. Moghaddam and M. R. Haghifam, "A Two Stage Model for Optimum Allocation of Electric Vehicle Parking Lots in Smart Grids," *2018 Smart Grid Conference (SGC)*, pp. 1-5, 2018.

[9] B. Zeng, Z. Zhu, H. Xu and H. Dong, "Optimal Public Parking Lot Allocation and Management for Efficient PEV Accommodation in Distribution Systems," *IEEE Transactions on Industry Applications*, vol. 56, no. 5, pp. 5984-5994, 2020.

[10] M. J. Mirzaei and A. Kazemi, "A dynamic approach to optimal planning of electric vehicle parking lots," *Sustainable Energy, Grids and Networks*, vol. 24, Oct. 2020.

[11] A. El-Zonkoly and L. dos santos Coelho., "Optimal allocation, sizing of PHEV parking lots in distribution system", International Journal of Electrical Power & Energy Systems, vol. 67, pp. 472-477, 2015.

[12] M. Rahmani-Andebili, H. Shen and M. Fotuhi-Firuzabad, "Planning and Operation of Parking Lots Considering System, Traffic, and Drivers Behavioral Model", *IEEE Transactions on Sytems, Man and Cybernetics: Sytems*, vol. 49, no. 9, pp. 1879 - 1892, 2018.

[۱۳] سمیه شمس‌الدینی لری، امیر عبداللهی، حسین هژبری، «جایابی احتمالاتی پارکینگ خودروهای برقی در شبکۀ توزیع با در نظر گرفتن

عدم قطعیت» مجله مهندسی و مدیریت انرژی دانشگاه کاشان، دوره ۹، شماره ۳، صفحات ۲۶-۳۹، ۱۳۹۸.

[14] Kazemi, M. A., Sedighizadeh, M., Mirzaei, M. J., Homaee, O., "Optimal siting and sizing of distribution system operator owned EV parking lots", *Applied Energy*, Vol. 179, pp. 1176-1184, 2016.

[۱۵] حسین کریمیان فرد، حسین حقیقت، «جایابی و اندازه بهینه ایستگاه‌های شارژ خودروهای الکتریکی بر اساس شاخص پایداری ولتاژ و کاهش تلفات شبکه توزیع»، نشریه انرژی ایران، دوره ۲۲، شماره ۲، صفحات ۱۲۱-۱۴۱، ۱۳۹۸.

[16] M. R. Mozafar, M. H. Moradi, M. H. Amini, "A simultaneous approach for optimal allocation of renewable energy sources and electric vehicle charging stations in smart grids based on improved GA-PSO algorithm," *Sustainable Cities and Society*, vol. 32, pp. 627-637, July. 2017.

[۱۷] محمدرضا آقاابراهیمی، مهدی تورانی، «مکان‌یابی و تعیین محدوده پارکینگ خودروهای الکتریکی با هدف بهبود قابلیت اطمینان شبکه و با در نظر گرفتن کاهش مسافت پیموده شده توسط خودروها»، مجله کیفیت و بهره‌وری صنعت برق ایران، دوره ۴، شماره ۸، صفحات ۲۷-۳۶، ۱۳۹۴.

[18] T. Sattarpour, M. Farsad, "Parking lot allocation with maximum economic benefit in a distribution network," *International Transactions on Electrical Energy Systems*, vol. 27, no. 1, Jan. 2017.

[19] E. Mohammadi, R. Fadaeinedjad, M. Rashidinejad and A. Mahdavinia, "Optimal placement and charge/discharge scheduling of electric parking lots considering reactive power generation capability," 2017 Smart Grid Conference (SGC), pp. 1-6, 2017.

[۲۰] حمید اصغری راد، میثم جعفری نوکندی، سیدمهدی حسینی، «تخصیص بهینه پارکینگ‌های شارژ خودروهای برقی در سیستم توزیع با در نظر گرفتن الگوی ترافیک شهری»، مجله مهندسی و مدیریت انرژی دانشگاه کاشان، دوره ۱۱، شماره ۲، صفحات ۷۰-۸۱، ۱۴۰۰.

[21] N. Neyestani, M. Y. Damavandi, G. Chicco and J. P. S. Catalão, "Effects of PEV Traffic Flows on the Operation of Parking Lots and Charging Stations," *IEEE Transactions on Smart Grid*, vol. 9, no. 2, pp. 1521-1530, Mar. 2018.

[22] CJ. Vidal, M Goetschalckx," A global supply chain model with transfer pricing and transportation cost allocation," *European Journal of Operational Research*, vol. 129, no. 1, pp 134-158, 2001.

[۲۳] محمد سعیدی مهرآباد، عادل اعظمی، «ارائه مدل بهینه‌سازی استوار دوسطحی در برنامه‌ریزی تولید با در نظر گیری تصمیمات قیمت‌گذاری به منظور پاسخگویی به تقاضا در فضای رقابتی: مطالعه موردی»، مجله پژوهش های مهندسی صنایع در سیستم‌های تولید، دوره ۵، شماره ۱۱، صفحات ۱۷۳-۱۹۱، ۱۳۹٦.

[24] W. H. Kersting, "Radial distribution test feeders, "*IEEE Transactions on Power Systems*," vol. 6, no. 3, pp. 975-985, Aug. 1991.

[25] K. J. Yanus. "Plug-in electric vehicle charging impacts on power systems," Master of Science in Department of Energy and Environment Division of Electric Power Engineering in Chalmers University of Technology. Göteborg, Sweden, 2010.

[26] A. Briones, J. Francfort, P. Heitmann, M. Schey, S. Schey, J. Smart, " Vehicle-to-grid (V2G) power flow regulations and building codes review by the AVTA", Idaho National Lab., Idaho Falls, ID, USA, 2012.

[27] S. Astroza, PC. Bhat, Chandra R. Bhat, RM. Pendyala, VM. Garikapati, " Understanding activity engagement across weekdays and weekend days: A multivariate multiple discrete-continuous modeling approach," *Journal of Choice Modelling*, vol. 28, pp. 56-70, 2018.

7. Appendix

7.1. Linear form of the product of two continuous variables

Assume that the product of two decision variables and y is z, and the upper and lower bounds for x and y are as follows:

$$x_{min} \le x \le x_{max} \quad \text{a-1}$$

$$y_{min} \le y \le y_{max} \quad \text{a-2}$$

Then it is possible to constrain z by the linear equations below:

$$x_{min} \cdot y \le z \le x_{max} \cdot y \quad \text{a-3}$$

$$y_{min} \cdot x \le z \le y_{max} \cdot x \quad \text{a-4}$$