Adaptive Robust H∞ finite-time congestion control design for TCP/AQM Network System with parametric uncertainties

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Abstract

The congestion control problem is one of the most essential subjects in the Transmission Control Protocol (TCP) Network because of the complex nonlinear model, uncertainties, and external disturbances. This paper extends the adaptive robust H∞ control finite-time approach to TCP network and presents a new solution to solve the congestion control problem employing Active Queue Management (AQM). Firstly, a modified nonlinear model of TCP network system with parametric uncertainties and external disturbance is given. Then, by several variable changes based on the backstepping method and Lyapunov function, adaptation and control laws were derived. Stability analysis is given to prove that all the signals of the closed-loop system are finite-time bounded. In addition, the results show that the proposed controller can guarantee both the transient and steady-state performance of the system, the queue of the TCP network system can track the desired queue and the disturbance is rejected satisfactory based on H∞ control part of the controller. Finally, a comparison example is considered to demonstrate the feasibility and superiority of the presented scheme.

Keywords

Adaptive control, Robust H∞ control, TCP/AQM, finite-time, parametric uncertainties.

1. Introduction

Internet traffic congestion has emerged as an important issue in communication network design because lack of significant attention to it can come into being some discomfort for our life. In [1], it was attempted to travel the trajectory of AQM research from 1993 with the first algorithm, Random Early Detection (RED) [2], to some works in 2011 [3]. The main reason for great attention to AQM is that when congestion of packets occurs at the outgoing queues in routers, it can be lead to poor performance, low reliability of the network, and Hopf bifurcation [4,5]. For example, authors in [4] analyze the problem of Hopf bifurcation control for a congestion control model in a wireless access network with time delay. It was shown when delay passes through a critical value, the Hopf bifurcation occurs and it might cause heavy oscillation and induce network instability. Several Internet congestion control approaches proposed that are mainly distributed algorithms to compete for sources so as not to exceed link capacities and hence avoid lockout behavior [6-9]. In comparison with passive queue management (PQM), AQM algorithms make sure that desired queue is tracked more precisely and enhance the performance of the network. After RED approach proposed in [2] by Floyd and Jacobson, many researchers introduced modified and improved AQM algorithms such as BLUE in [10] and REM in [11]. However, RED and its variant algorithms are too sensitive to parameter configuration, which makes parameter tuning complex and difficult. In order to design and understand the behavior of internet systems better and avoid the above-mentioned, in [12-14] a model of TCP dynamical behavior was derived by using the theory of fluid-flow theory and some assumptions. The mathematical model of TCP/AQM developed in recent years and combining these models with some traditional and advanced control methods, a great set of AQM algorithms were proposed such as PI [15], PI-PD [16], PID [17], sliding mode [18], adaptive integral backstepping [19,20], feed-forward neural networks [21] and adaptive fuzzy method [22].

It is well known that robust and adaptive control techniques are some effective tools to deal with uncertainties and external disturbances [23,24]. Among the approaches to handle external disturbances, H∞ robust control method is a powerful technique [25,26]. On the other hand, in many real systems, it is necessary to obtain the errors or states of the system identically equal to zero in a finite time (FT) [27-30]. Compared with the infinite-time results, FT control ones have some additional advantages such as fast convergence, better robustness, and anti-interference ability. Therefore, many scholars focus on two issues; finite-time stability (FTS) and finite-time control problem (FTC), and try to extend some methods of solving the infinite-time problem to study the finite-time case. One of the first analyses for FTS was given in [27] for continuous autonomous systems. When uncertainty and disturbance are included in the nonlinear system simultaneously and FTS conditions need to be derived, the problem is more complex but the result can be used for a wide range of practical applications. In [29], FT adaptive robust control problem for a class of nonlinear time-delay systems studied with external disturbance via the Lyapunov-Krasovskii (L-K) method and presents some delay-independent and delay-dependent results. In [31], the observer-based finite-time robust control problem of a class of nonlinear time-delay systems via the Hamiltonian function method investigated and proposed some finite-time robust stabilization results. Authors in [32], based on the comparison principle and the notions of FT stable and FT escaping functions, established some Lyapunov-like theorems for testing both local and global FTS. FTS and FT boundedness (FTB) of fractional order switched systems are investigated in [33]. By employing the average dwell time technique and Lyapunov functional method, some sufficient conditions for FTS and FTB of fractional order switched systems are proposed in [33].

During the last decade, a lot of research works to control and especially FTC of TCP/AQM system have been attracted by adaptive and robust approaches such as [19,22,34-38]. For example, adaptive FT congestion control of TCP/AQM utilizing the funnel control, RBFNNs for approximating of uncertain functions and, sliding mode control (SMC) designed in [34] and ensured that the tracking error converges to the prescribed boundary in finite-time. In [35], the issue of adaptive practical FT congestion control for the (TCP/AQM) network with unknown hysteresis and external disturbance is investigated. In [19] an adaptive tracking controller is proposed to deal with the congestion problem in a wireless network. The uplink and downlink packet losses are estimated by adaptive update laws. In [22], a performance constraint control problem is designed for TCP/AQM system with external disturbance and input saturation by the basis of the backstepping-like design procedure and fuzzy approximation technique. It was shown that the proposed adaptive fuzzy controller with the prescribed constraint ensures that the transient and steady-state performances of the tracking errors can be satisfied. Authors in [36], employing the backstepping technique, finite-time control method, and H∞ control theory, a robust H∞ finite-time controller proposed to guarantee the finite-time convergence of the queue tracking error. The prescribed performance control (PPC) technique is extended to TCP/AQM network congestion problem in [37], and a congestion control algorithm is presented to solve a tracking problem by using PPC, backstepping, adaptive, and H∞ control. In [37] the available link capacity (C) was assumed to be uncertain and asymptotic stability in the lack of disturbance input was guaranteed.

To the best of the author’s knowledge, there is no paper addressing the adaptive robust H∞ finite-time congestion control design of TCP/AQM. It is assumed that the TCP/AQM model included two uncertain parameters as well as an external disturbance. Using the Lyapunov function, adaptive laws have been extracted such that the closed-loop system is FTS and guaranteed the queue tracking error converges to zero in finite time. Moreover, sufficient conditions have been derived for satisfying the H∞ performance index to overcome the effect of external disturbance input. The TCP/AQM model investigated in this paper is similar to [34]. However, in this paper Lyapunov approach is utilized to establish adaptive laws and the robustness of the closed-loop system is satisfied by the H∞ method. Compared with [36], in this paper, adaptive control design is added to deal with parametric uncertainties. In comparison with [37], the unknown parameters considered in this paper and the method of adaptive technique are different from the investigated model and approach in [37]. Also, in this paper, FTC conditions derived which can guarantee the states and error signals tend to zero in the finite time while in [37] asymptotical stability has been achieved. Recently, in [39], a new AQM model for Internet chaotic behavior using Petri Nets has been proposed. The problem of input-saturation in the design of adaptive controller for TCP/AQM system is considered in [40]. The use of neural networks and improved proportional–integral controller has been reported in very recent works [41] and [42], respectively. In summary, the innovations of this paper in comparison with previous works, especially [34-42] are:

1- The combination of the adaptive control method and robust $H\infty$ control approach with consideration of parametric uncertainties

2- Using limited time stability instead of asymptotic stability speeds up the system in response.

3- Defining a Lyapunov function and new adaptation rules to achieve the intended goals.

The remainder of this paper is structured as follows. Firstly, a model of TCP/AQM as well as some preliminaries and necessary definitions are presented in Section 2. The proposed controller design and main results are developed in Section 3. In section 4, simulation results are provided to show the effectiveness and feasibility of the presented approach. Finally, this paper ends with a conclusion and cited references.

2. Problem formulation

2.1. TCP/AQM Model

Inspired by [34], we consider the following TCP/AQM network model to show the behavior of the window size of senders and the queue length at the router.

$$\begin{cases} \dot{W}(t) = \dfrac{\alpha}{R(t)} - \dfrac{2(1-\beta)W(t)}{1+\beta}\dfrac{W(t)}{R(t)}p(t) \\ \dot{q}(t) = \dfrac{N(t)}{R(t)}W(t) - C(t) + \omega(t) \\ R(t) = \dfrac{q(t)}{C(t)} + T_P \end{cases} \tag{1}$$

Where $W(t)$ is the average size of the TCP window (packets), $q(t)$ is the average size of queue length and $R(t)$ is the round-trip time. $p(t)$ is the probability of packet loss, which satisfies that $p(t) \in [0,1]$. α, β are some unknown parameters of the system, which satisfy $\alpha > 0, \beta \in [0,1]$. $N(t)$ represents the number of TCP sessions, T_p is propagation delay and $C(t)$ is the queue capacity. $\omega(t)$ is the external disturbance.

Remark 1: Because of the fact that most Internet scenarios are constant over time or time-varying very slowly, the parameters $N(t)$ and $C(t)$ can be considered as fixed values like [34,35]. Therefore, $N(t)$ and $C(t)$ can be rewritten as N and C.

Remark 2: The presented model in (1) is more general in comparison with [20] and [36]. Because if parameters α,β are set to 1 and 0, respectively, in (1), the model in [36] is achieved, and if set to 1 and 0.6, respectively, the model in [20] is achieved.

According to [34,35,37], the rate is defined as follows:

$$r(t)=\frac{W(t)}{R(t)} \tag{2}$$

and its time derivative can be calculated as

$$\dot{r}(t)=\frac{\dot{W}(t)R(t)-W(t)\dot{R}(t)}{R^2(t)} \tag{3}$$

In addition, we have

$$\dot{R}(t)=\frac{1}{C}\dot{q}(t) \tag{4}$$

Using (2), the second equation in (1) can be rewritten as

$$\dot{q}(t)=Nr(t)-C+\omega(t) \tag{5}$$

From (1), (2) and (3), one can obtain the following relation

$$\dot{r}(t)=\frac{\alpha}{R^2(t)}-\frac{2(1-\beta)W(t)}{1+\beta}\frac{W(t)}{R^2(t)}p(t)-\frac{W(t)}{R^2(t)}\frac{1}{C}\big(Nr(t)-C+\omega(t)\big)=\frac{\alpha}{R^2(t)}+\frac{r(t)}{R(t)}-\frac{Nr^2(t)}{R(t)C}-\frac{2(1-\beta)r^2(t)}{1+\beta}p(t)-\frac{r(t)}{R(t)C}\omega(t) \tag{6}$$

Set $x_1(t)=q(t)$, $x_2(t)=r(t)$,$u(t)=p(t)$ then the state space equations can be written as

$$\begin{cases}\dot{x}_1(t)=Nx_2(t)-C+\omega(t)\\ \dot{x}_2(t)=\mu_1\varphi_1(x_1,x_2)+\mu_2\varphi_2(x_1,x_2)u(t)+\\ \quad\Psi(x_1,x_2)+\phi(x_1,x_2)\omega(t)\end{cases} \tag{7}$$

Where

$$\mu_1=\alpha\ ,\qquad \mu_2=\frac{1-\beta}{1+\beta} \tag{8}$$

And

$$\varphi_1(x_1,x_2)=\frac{1}{R^2(t)}=\frac{1}{\left(\frac{1}{C}x_1(t)+T_P\right)^2} \tag{9}$$

$$\varphi_2(x_1,x_2)=-2{x_2}^2(t) \tag{10}$$

$$\Psi(x_1,x_2)=\frac{r(t)}{R(t)}-\frac{Nr^2(t)}{R(t)C}=\frac{x_2(t)}{\frac{1}{C}x_1(t)+T_P}\left(1-\frac{N}{C}x_2(t)\right) \tag{11}$$

$$\phi(x_1,x_2)=-\frac{r(t)}{R(t)C}=-\frac{x_2(t)}{x_1(t)+CT_P} \tag{12}$$

Assumption 1: Since α and β are unknown parameters, μ_1 and μ_2 are also uncertain constants. Moreover, it is assumed that the sign of μ_2 is specified and $0<\sigma_1\le|\mu_2|\le\sigma_2$. Because $\beta\in[0,1]$, it is obvious that the sign of μ_2 is known, but we aim to propose a controller for system (7) in general.

Assumption 2: To guarantee that input control satisfied the condition $p(t)\in[0,1]$, the saturation function is introduced as follows:

$$u(t)=sat(v)=\begin{cases}0 & v<0\\ v & 0\le v\le 1\\ 1 & v>1\end{cases} \tag{13}$$

The following approximation is employed to avoid the sharp corners of $sat(v)$:

$$u(t)=f(t)+\Delta(t) \tag{14}$$

where $f(t)=tanh(v)$ and $\Delta(t)=sat(v)-f(t)$.

2.2. Preliminaries

In this subsection, some definitions and basic results are given, which will be used in this paper.

Definition 1 [36]: Consider a nonlinear control system with

$$\begin{cases}\dot{X}=f(X)+g(X)u+d(X)\omega\\ Y=h(X)\end{cases} \tag{15}$$

where $u\in\mathbb{R}^m$, $X\in\mathbb{R}^n$, $\omega\in\mathbb{R}^r$ and $Y\in\mathbb{R}^q$ denote the input, the state, the disturbance and the output of the system, respectively. System (15) is said to be $H\infty$ globally asymptotically stabilizable, if there exists a state feedback $u=K(X)$ such that the closed-loop system

$$\begin{cases}\dot{X}=f(X)+g(X)K(X)+d(X)\omega\\ Y=h(X)\end{cases} \tag{16}$$

(i) is globally asymptotically stable, that is, $\|X\|$ approaches zero as t goes to infinity when $\omega=0$; **(ii)** has an L2 gain less than or equal to a positive constant γ for all $t_1>t_0$ and all disturbances $\omega\in L_2$; that is, the following inequality holds for $X(t_0)=0$.

$$\int_{t_0}^{t_1}\|Y\|^2dt\le\gamma^2\int_{t_0}^{t_1}\|\omega\|^2dt \tag{17}$$

Lemma 1 [34]: Assume that there exists a continuous differentiable function $V(x(t))>0$ which satisfies the following inequality:

$$\dot{V}(x(t))+cV(x(t))^{\mu}\le 0\ ,\qquad \forall\, t>t_0 \tag{18}$$

Where $c>0$ *and* $0<\mu<1$. Then, system (16) with $\omega=0$ and $u=k(x)$ is finite-time stable. The settling time can be calculated by

$$T\le\frac{V(x(t_0))^{1-\mu}}{c(1-\mu)} \tag{19}$$

3. Main Results

The control goal of this paper is to construct a control law $u(x_1,x_2)$ such that the queue $x_1(t)=q(t)$ tracks the target queue length q_d from any initial conditions in finite-time. Since the parameters μ_1 ,μ_2 are unknown, adaptive laws are introduced and combined with a finite-time $H\infty$ design for the TCP/AQM nonlinear system with the external disturbances in (7). In order to make the TCP/AQM nonlinear system finite-time stable, the following virtual controllers are defined.

$$\begin{cases} \xi_1(z_1) = -\rho_1 z_1 - \rho_2 |z_1|^{\mu} + \dot{q}_d - \dfrac{z_1}{\gamma^2} \\ \xi_2(z_2) = -\rho_3 z_2 - \rho_4 |z_2|^{\mu} \\ \qquad - \dfrac{\rho_5 |\hat{\mu}_1|^{\mu+1} + \rho_6 |\hat{\mu}_2|^{\mu+1}}{z_2} \end{cases} \tag{20}$$

Where $\rho_i > 0$ $(i = 1, \ldots, 6)$, $0 < \mu < 1$ and error variables are defined as

$$z_1 = x_1 - q_d \ , \ z_2 = N x_2 - C - \xi_1(z_1) \tag{21}$$

and $\hat{\mu}_1 , \hat{\mu}_2$ are the estimation of μ_1 , μ_2, respectively, which introduced later.

The derivative of z_1 , z_2 in (21) expressed as

$$\dot{z}_1 = \dot{x}_1 - \dot{q}_d = N x_2 - C + \omega - \dot{q}_d \tag{22}$$

$$\begin{aligned} \dot{z}_2 &= N \dot{x}_2 - \dot{\xi}_1(z_1) \\ &= N\mu_1 \varphi_1(x_1, x_2) + N\mu_2 \varphi_2(x_1, x_2) u \\ &+ N\Psi(x_1, x_2) + N\phi(x_1, x_2)\omega - \dot{\xi}_1(z_1) \end{aligned} \tag{23}$$

The derivative of $\xi_1(z_1)$ in (20) can be written as

$$\begin{aligned} \dot{\xi}_1(z_1) &= -\rho_1 \dot{z}_1 - \rho_2 \dot{z}_1 sgn(z_1) |z_1|^{\mu-1} + \ddot{q}_d \\ &- \frac{\dot{z}_1}{\gamma^2} \dot{z}_1 \left(-\rho_1 - \rho_2 sgn(z_1) |z_1|^{\mu-1} - \frac{1}{\gamma^2} \right) + \ddot{q}_d \\ &= \dot{z}_1 \chi + \ddot{q}_d \end{aligned} \tag{24}$$

Where the notation sgn is the well-known sign function and

$$\chi = -\rho_1 - \rho_2 sgn(z_1) |z_1|^{\mu-1} - \frac{1}{\gamma^2} \tag{25}$$

Theorem 1: For a given TCP/AQM nonlinear system (7), a finite-time adaptive robust $H\infty$ control law as follows

$$\begin{aligned} u = \frac{1}{N\hat{\mu}_2 \varphi_2(x_1, x_2)} \Bigg(\hat{\mu}_2 sign(\mu_2) \Bigg(\xi_2(z_2) \\ - \frac{(N\phi(x_1, x_2) - \chi)^2 z_2}{\sigma_1 \gamma^2} \Bigg) \\ + (N x_2 - C - \dot{q}_d)\chi + \ddot{q}_d \\ - z_1 - N\hat{\mu}_1 \varphi_1(x_1, x_2) \\ - N\Psi(x_1, x_2) \Bigg) \end{aligned} \tag{26}$$

With adaptive laws as

$$\begin{cases} \dot{\hat{\mu}}_1 = \Gamma_1 \left(z_2 N \varphi_1(x_1, x_2) \right) \\ \dot{\hat{\mu}}_2 = \Gamma_2 \left(-\dfrac{k_1 z_2}{\hat{\mu}_2} - \dfrac{\hat{\mu}_1}{\hat{\mu}_2} N \varphi_1(x_1, x_2) z_2 \right) \end{cases} \tag{27}$$

Where $z_1, z_2 , \chi, \xi_1(z_1), \xi_2(z_2)$ are introduced in (20), (24) and (19), respectively and $\rho_1 > \frac{1}{2} r_1^2, \rho_3 > \frac{1}{2\sigma_1} r_2^2 , \Gamma_1 > 0$, $\Gamma_2 > 0$ guarantees globally finite-time stability of closed-loop system for $\omega = 0$, and $H\infty$ tracking performance index with prescribed attenuation level γ in (19) with $Y = [r_1 z_1 \quad r_2 z_2]^T$. Fig.1 shows the block diagram for the proposed control method. Moreover, the settling time can be obtained by

$$T \leq \frac{V(x(t_0))^{1-\partial}}{\rho(1-\partial)} \tag{28}$$

Where

$$\begin{aligned} \rho &= min \left\{ \begin{matrix} \rho_2 2^{\partial}, \rho_4 \sigma_1 2^{\partial}, \rho_5 \sigma_1 (2\Gamma_1)^{\partial} \\ , \rho_6 \sigma_1 (2\Gamma_2)^{\partial} \end{matrix} \right\}, \\ \partial &= \frac{\mu + 1}{2} \end{aligned} \tag{29}$$

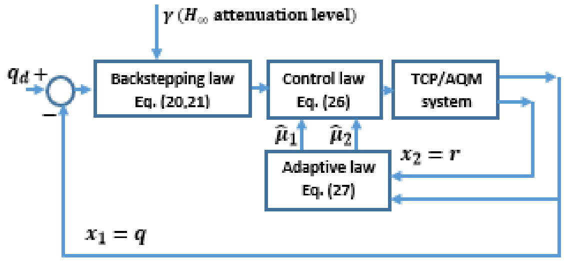


Fig. 1. Block diagram of the proposed method

Proof:

The Lyapunov function candidate chosen as follows

$$V = V_1 + V_2 + V_3 \tag{30}$$

Where

$$\begin{aligned} V_1 &= \frac{1}{2} z_1^2 , V_2 = \frac{1}{2} z_2^2 , \\ V_3 &= \frac{1}{2} \Gamma_1^{-1} \tilde{\mu}_1^2 + \frac{1}{2} \Gamma_2^{-1} \tilde{\mu}_2^2 \end{aligned} \tag{31}$$

and $\tilde{\mu}_1 = \hat{\mu}_1 - \mu_1$, $\tilde{\mu}_2 = \hat{\mu}_2 - \mu_2$ are the estimation error of unknown parameters μ_1 and μ_2. The derivative of V_1 and V_2 along the trajectories of the system can be written as

$$\begin{aligned} \dot{V}_1 &= z_1 \dot{z}_1 = z_1 (N x_2 - C + \omega - \dot{q}_d) \\ &= z_1 (z_2 + \xi_1(z_1) + \omega - \dot{q}_d) \\ &= z_1 \left(\begin{matrix} z_2 + \left(-\rho_1 z_1 - \rho_2 |z_1|^{\mu} + \dot{q}_d - \dfrac{z_1}{\gamma^2} \right) \\ + \omega - \dot{q}_d \end{matrix} \right) \\ &= z_1 \left(\begin{matrix} z_2 - \rho_1 z_1 - \rho_2 |z_1|^{\mu} \\ - \dfrac{z_1}{\gamma^2} + \omega \end{matrix} \right) \end{aligned} \tag{32}$$

$$\begin{aligned} \dot{V}_2 &= z_2 \left(\begin{matrix} N\mu_1 \varphi_1(x_1, x_2) + N\mu_2 \varphi_2(x_1, x_2) u \\ + N\Psi(x_1, x_2) + N\phi(x_1, x_2)\omega \\ - \dot{\xi}_1(z_1) \end{matrix} \right) \\ &= z_2 \left(\begin{matrix} N\mu_1 \varphi_1(x_1, x_2) + N\mu_2 \varphi_2(x_1, x_2) u \\ + N\Psi(x_1, x_2) + N\phi(x_1, x_2)\omega \\ -(N x_2(t) - C + \omega - \dot{q}_d)\chi - \ddot{q}_d \end{matrix} \right) \end{aligned} \tag{33}$$

The summation of (32) and (33) results that

$$\begin{aligned} \dot{V}_1 + \dot{V}_2 &= z_1 \left(-\rho_1 z_1 - \rho_2 |z_1|^{\mu} - \frac{z_1}{\gamma^2} + \omega \right) \\ &+ z_2 \left(\begin{matrix} z_1 + N\mu_1 \varphi_1(x_1, x_2) \\ + N\mu_2 \varphi_2(x_1, x_2) u + N\Psi(x_1, x_2) \\ + (N\phi(x_1, x_2) - \chi)\omega \\ -(N x_2 - C - \dot{q}_d)\chi - \ddot{q}_d \end{matrix} \right) \end{aligned} \tag{34}$$

By substituting the control law (26) in (34), we have

$$\begin{aligned}&\dot{V}_1+\dot{V}_2\\&=z_1\left(-\rho_1 z_1-\rho_2|z_1|^{\mu}-\frac{z_1}{\gamma^2}+\omega\right)\\&+z_2\begin{pmatrix}k_1\left(1-\frac{\mu_2}{\hat{\mu}_2}\right)\\+N\varphi_1(x_1,x_2)\left(\mu_1-\hat{\mu}_1\frac{\mu_2}{\hat{\mu}_2}\right)\\+k_2|\mu_2|\\+(N\phi(x_1,x_2)-\chi)\omega\end{pmatrix}\end{aligned} \quad (35)$$

Where $k_1=z_1+N\Psi(x_1,x_2)-(Nx_2-C-\dot{q}_d)\chi-\ddot{q}_d$ and $k_2=\xi_2(z_2)-\frac{(N\phi(x_1,x_2)-\chi)^2 z_2}{\sigma_1\gamma^2}$.

The derivative of V_3 can be expressed as

$$\begin{aligned}\dot{V}_3&=\Gamma_1^{-1}\tilde{\mu}_1\dot{\hat{\mu}}_1+\Gamma_2^{-1}\tilde{\mu}_2\dot{\hat{\mu}}_2\\&=\Gamma_1^{-1}(\hat{\mu}_1-\mu_1)\dot{\hat{\mu}}_1+\Gamma_2^{-1}(\hat{\mu}_2-\mu_2)\dot{\hat{\mu}}_2\end{aligned} \quad (36)$$

Substituting adaptive laws (27) in (36) leads to the following equation

$$\begin{aligned}\dot{V}_3&=\Gamma_1^{-1}(\hat{\mu}_1-\mu_1)\Gamma_1\big(z_2N\varphi_1(x_1,x_2)\big)\\&+\Gamma_2^{-1}(\hat{\mu}_2-\mu_2)\Gamma_2\left(-\frac{k_1z_2}{\hat{\mu}_2}-\frac{\hat{\mu}_1}{\hat{\mu}_2}z_2N\varphi_1(x_1,x_2)\right)\\&=\hat{\mu}_1z_2N\varphi_1(x_1,x_2)-\mu_1z_2N\varphi_1(x_1,x_2)\\&-\frac{k_1z_2}{\hat{\mu}_2}(\hat{\mu}_2-\mu_2)-\hat{\mu}_1z_2N\varphi_1(x_1,x_2)\\&+\mu_2\frac{\hat{\mu}_1}{\hat{\mu}_2}z_2N\varphi_1(x_1,x_2)\end{aligned} \quad (37)$$

Sum of the equations (35) and (37) leads to

$$\begin{aligned}\dot{V}=\dot{V}_1+\dot{V}_2+\dot{V}_3&=z_1\begin{pmatrix}-\rho_1z_1-\rho_2|z_1|^{\mu}\\-\frac{z_1}{\gamma^2}+\omega\end{pmatrix}\\&+z_2(k_2|\mu_2|+(N\phi(x_1,x_2)-\chi)\omega)\end{aligned} \quad (38)$$

By and substituting k_2 and $\xi_2(z_2)$ from equation (20), the second part of equation (38) can be rewritten as follows

$$\begin{aligned}&z_2(k_2|\mu_2|+(N\phi(x_1,x_2)-\chi)\omega)\\&=z_2\left(|\mu_2|\begin{pmatrix}-\rho_3z_2-\rho_4|z_2|^{\mu}\\-\frac{\rho_5|\hat{\mu}_1|^{\mu+1}+\rho_6|\hat{\mu}_2|^{\mu+1}}{z_2}\\-\frac{(N\phi(x_1,x_2)-\chi)^2z_2}{\sigma_1\gamma^2}\end{pmatrix}+(N\phi(x_1,x_2)-\chi)\omega\right)\end{aligned} \quad (39)$$

To prove the $H\infty$ tracking performance within the prescribed attenuation level γ, consider the following function

$$H=\dot{V}+\frac{1}{2}(\|z\|^2-\gamma^2\|\omega\|^2) \quad (40)$$

$$=\dot{V}_1+\dot{V}_2+\dot{V}_3+\frac{1}{2}r_1^2z_1^2+\frac{1}{2}r_2^2z_2^2-\frac{1}{2}\gamma^2\omega^2$$

By the combination of equations (38)-(40), the following equation can be obtained:

$$\begin{aligned}H&=-\left(\rho_1-\frac{1}{2}r_1^2\right)z_1^2-\left(\rho_3|\mu_2|-\frac{1}{2}r_2^2\right)z_2^2\\&-\rho_2|z_1|^{\mu+1}-\frac{z_1^2}{\gamma^2}+z_1\omega-\rho_4|\mu_2||z_2|^{\mu+1}\\&-|\mu_2|\frac{(N\phi(x_1,x_2)-\chi)^2z_2^2}{\sigma_1\gamma^2}\\&+z_2(N\phi(x_1,x_2)-\chi)\omega-\frac{1}{4}\gamma^2\omega^2-\frac{1}{4}\gamma^2\omega^2\\&-\rho_5|\mu_2||\hat{\mu}_1|^{\mu+1}-\rho_6|\mu_2||\hat{\mu}_2|^{\mu+1}\\&\le-\rho_2|z_1|^{\mu+1}-\rho_4\sigma_1|z_2|^{\mu+1}-\left(\frac{z_1}{\gamma}-\frac{\gamma\omega}{2}\right)^2\\&-\left(\frac{(N\phi(x_1,x_2)-\chi)z_2}{\gamma}-\frac{\gamma\omega}{2}\right)^2\\&-\rho_5\sigma_1|\hat{\mu}_1|^{\mu+1}-\rho_6\sigma_1|\hat{\mu}_2|^{\mu+1}\\&\le-\rho_2|z_1|^{\mu+1}-\rho_4\sigma_1|z_2|^{\mu+1}\\&-\rho_5\sigma_1|\tilde{\mu}_1|^{\mu+1}-\rho_6\sigma_1|\tilde{\mu}_2|^{\mu+1}\\&\le-\rho_22^{\frac{\mu+1}{2}}\left(\frac{z_1^2}{2}\right)^{\frac{\mu+1}{2}}-\rho_4\sigma_12^{\frac{\mu+1}{2}}\left(\frac{z_2^2}{2}\right)^{\frac{\mu+1}{2}}\\&-\rho_5\sigma_1(2\Gamma_1)^{\frac{\mu+1}{2}}\left(\frac{1}{2}\Gamma_1^{-1}\tilde{\mu}_1^2\right)^{\frac{\mu+1}{2}}\\&-\rho_6\sigma_1(2\Gamma_2)^{\frac{\mu+1}{2}}\left(\frac{1}{2}\Gamma_2^{-1}\tilde{\mu}_2^2\right)^{\frac{\mu+1}{2}}\le-\rho V^{\partial}\end{aligned} \quad (41)$$

where ρ and ∂ introduced in (29). It can be concluded from (39) that if $\omega(t)=0$ then $\dot{V}+\rho V^{\partial}\le-\frac{1}{2}\|z\|^2<0$, which based on Lemma 1 gaurantees finite-time stability of closed loop system (7). Also, if $\omega(t)\neq 0$ (39), by integrating both sides of $H=\dot{V}+\frac{1}{2}(\|z\|^2-\gamma^2\|\omega\|^2)\le 0$ from t_0 to t_1, we have $V(t_1)-V(t_0)+\frac{1}{2}\int_{t_0}^{t_1}\|z\|^2-\gamma^2\|\omega\|^2\le 0$. Because V is a nonincreseing function we have $V(t_1)\le V(t_0)$ and this means that $\frac{1}{2}\int_{t_0}^{t_1}\|z\|^2-\gamma^2\|\omega\|^2\le 0$ which results that the condition (17) is satisfied. Therefore, the closed loop system is $H\infty$ finite-time stable with the attenuation prescribed level γ with control law (26) and adaptive laws (27). This completes the proof. ■

Theorem 1 shows the main idea of this paper which is employing the new adaptive control technique to overcome parametric uncertainties in the model of nonlinear TCP/AQM system as well as $H\infty$ approach to reduce the effect of disturbance with attenuation level rejection γ. For nonlinear systems such as the congestion control problem which is considered in this paper, solving the basic optimization $H\infty$ problem is a difficult issue. Because of using nonlinear adaptive laws, the problem is more challenging. Therefore, in this paper, by designing an adaptive state feedback controller, stability of the closed-loop system is obtained and it is guaranteed that the gain from disturbances to the tracking error smaller than or equal to a given positive number related to the prescribed performance level. Compared with [22], in this paper robust $H\infty$ part is added to the controller. In comparison with [36,37], parametric uncertainties are also considered in the model and the designed controller is more applicable. Therefore, the novel adaptive and robust control law not only can cover the advantages of previous works but also overcomes parametric uncertainties and external disturbances in a finite time.

Remark3: As can be seen from Equation (27), $\hat{\mu}_2$ as one of the parameters of the adaptive controller is at the denominator, and if this parameter tends to zero during the adaptation, it can cause problems in the implementation and simulation of the closed-loop control system. Several solutions have been proposed to prevent this, including defining a threshold for the parameters and stopping the adaptation operation if the parameter enters a certain range. Another operational solution is to stop the adaptation operation if the system output error is at a certain limit. In this paper, the latter method is used, in such a way that if the condition $|z_1| = |x_1 - q_d| \leq 0.001$ is met, the values of $\dot{\hat{\mu}}_1, \dot{\hat{\mu}}_2$ are considered equal to zero, and as a result, the adaptation operation is stopped. It can be clearly seen in the simulation results that after placing z_1 in the above range, the matching parameters have found a constant value and the matching operation has stopped.

4. Simulation Results

Fig. 2 shows a network dumbbell topology structure with a single bottleneck link. Matlab software is utilized to test the effectiveness of the presented method. Table I. includes the parameters of the TCP/AQM system are provided as follows [36] and the control parameters which is needed for the proposed method. We consider three sets of values for uncertain parameters α, β as 3 scenarios specified by the index and are given in Table 2. The first scenario with values $\alpha_1 = 1, \beta_1 = 0$, is also considered in [34,36], but for the others, only the proposed approach in this paper provided a solution in the adaptive robust structure.

Table I. Controller parameters and fixed system parameters

Controller parameters	Fixed system parameters
$\Gamma_1 = \Gamma_2 = 0.8$	$TP = 0.1s$
$\rho_1 = \rho_2 = 8$	$N = 60$
$\rho_3 = 20$ $\rho_4 = 15$	$C = 3000\frac{pakets}{s}$
$\rho_5 = \rho_6 = 5$	$q_d = 100$
$\gamma = 0.2, \mu = 0.9$	$q(0) = 100.2$

Table II. 3 scenarios for uncertain parameters

Scenario number	Uncertain system parameters
1	$\alpha_1 = 1, \beta_1 = 0$
2	$\alpha_2 = 0.5, \beta_2 = 0.5$
3	$\alpha_3 = 0.6, \beta_3 = 0.3$

The external disturbance $\omega(t)$ is considered as two following chosen functions which the only first one, $\omega_1(t)$, is also applied to the system in [34,36].

$$\begin{aligned} \omega_1(t) &= 0.2e^{-0.5t} \quad \forall\, t \geq 0 \\ \omega_2(t) &= sin(t) \quad \forall\, t \in [1,1.5] \end{aligned} \tag{42}$$

Simulation results are collected in Figs. 3-7. Fig. 3 shows the results of applying the methods FTSMC, TSMC, SMC, and the proposed method in [34] to the queue length ($q(t)$) in the TCP / AQM system. These controllers are designed with the assumption that the values of α and β are equal to 1 and 0 (the first scenario in Table 2), respectively and external disturbance is assumed to be in the form of $\omega_1(t)$.

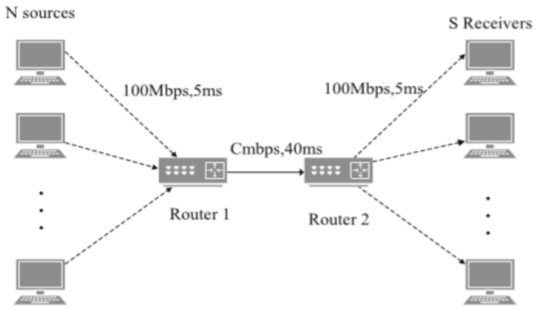
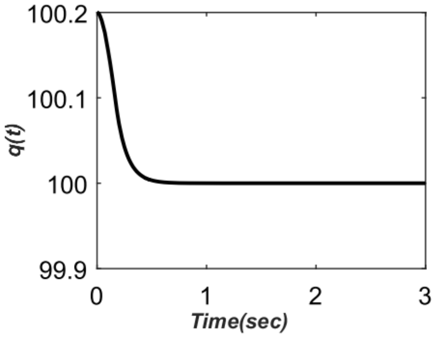


Fig. 2. The network topology [34].

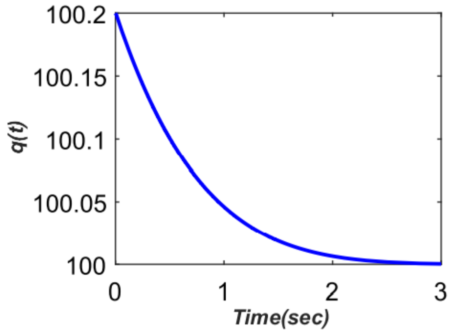
The result of the design of a finite-time robust $H\infty$ adaptive controller in this paper for the first scenario is also shown in Fig. 4. From viewing the figures presented in Fig. 3, it can be concluded that methods FTSMC and [34] have achieved almost the same results in terms of response speed. TSMC method, however, shows a sensible reduction in response speed. SMC method causes some fluctuations at the beginning of the response, while the time to reach the final value is longer compared to the previous 3 methods. The result of applying the method proposed in this paper for the first scenario shown in Fig. 4, shows that this method has been able to show a better transient response compared to the previous 4 methods and queue length move much faster towards the desired value. Although, the time required to eliminate the steady-state error in it is approximately equal to methods FTSMC and [34]. But one of the most important achievements of this paper is that the proposed method has been able to overcome the uncertainties that may appear in parameters α and β. The new values for these two parameters are given in Scenarios 2 and 3 in Table II. It should be noted that in previous methods, the above parametric uncertainties have not been considered in the design Figures 2 and 3 show the queue length ($q(t)$), control input ($u(t)$) and adaptive parameters ($\hat{\mu}_1$ *and* $\hat{\mu}_2$) curves, for the values of $\alpha_2 = 0.5$, $\beta_2 = 0.5$ (scenario 2) and $\alpha_3 = 0.6, \beta_3 = 0.3$ (scenario 3), respectively. In Fig. 5 (a) and Fig. 6 (a), the $q(t)$ are shown for scenarios 2 and 3. It can be seen that the system response has not changed much even in the presence of parametric uncertainties and the controller has been able to eliminate the effect of these uncertainties well. Although there are strong fluctuations in the input signal and adaptive parameters, but because there are no physical actuator in this system, the system can tolerate these fluctuations. In order to demonstrate the system's ability to eliminate the effect of oscillating perturbations, disturbance $\omega_2(t)$, which has a sinusoidal form, is considered and the results for $q(t)$, $u(t)$, $\hat{\mu}_1$, and $\hat{\mu}_2$ are shown in Fig. 7. It can be seen that although the amplitude of the sinusoidal perturbation applied in the considered time interval was 1, but its effect on $q(t)$ was only 0.05 and this indicates that the robust $H\infty$ part of the controller was able to eliminate the sinusoidal external disturbance effect. The conclusion of the simulation section is that the proposed controller has the ability to eliminate the effect of parametric uncertainties and different types of perturbations.

5. Conclusion

In this paper, an uncertain TCP/AQM model with parametric uncertainties has been considered, and by introducing a general form for state-space equations a novel adaptive congestion controller has been designed based on Lyapunov function, finite-time lemma, and H∞ theory. The developed controller can guarantee that the queue length tracks, in the finite-time, the desired queue length in the presence of parametric uncertainties. Besides, by comparing with the existing control methods, it was shown that the settling time to track the desired queue is reduced by the proposed controller and it can attenuate the influence of the external disturbances to a given degree. Finally, simulation results show that the proposed algorithm can offer a better tracking performance, the effectiveness, and the superiority of the proposed approach.



(a)



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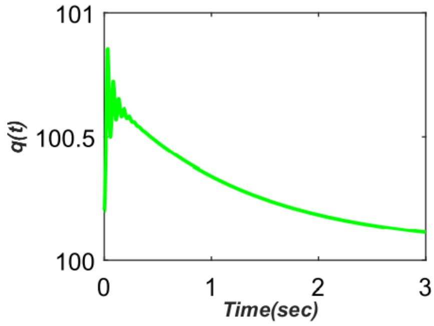

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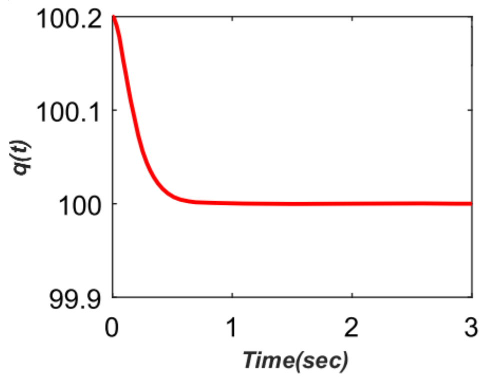


(d)

Fig. 3. The queue length $q(t)$ (a) FTSMC, (b) TSMC, (c) SMC and (d) proposed method in [34] for $\alpha_1 = 1,\ \beta_1 = 0$ and the external disturbance $\omega_1(t)$ (All figures from [34]).

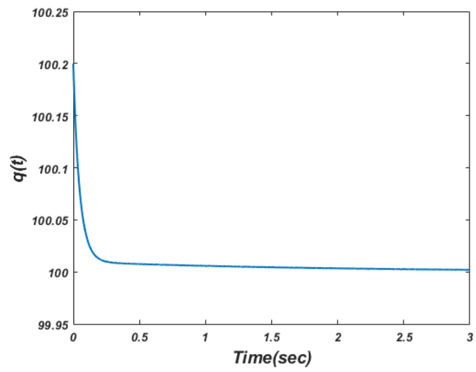
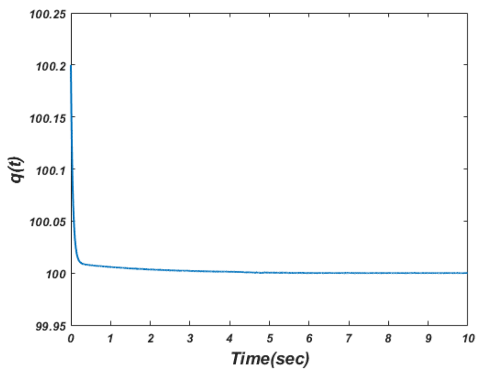
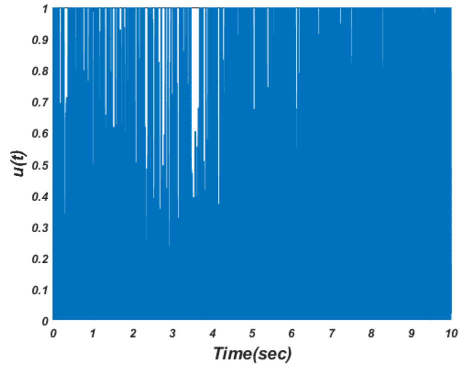


Fig. 4. The queue length $q(t)$ of the proposed method in this paper for $\alpha_1 = 1,\ \beta_1 = 0$ and the external disturbance $\omega_1(t)$.



(a)



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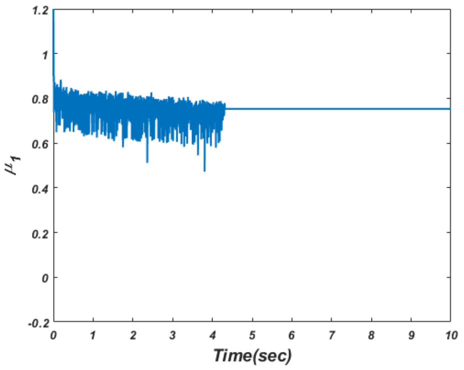

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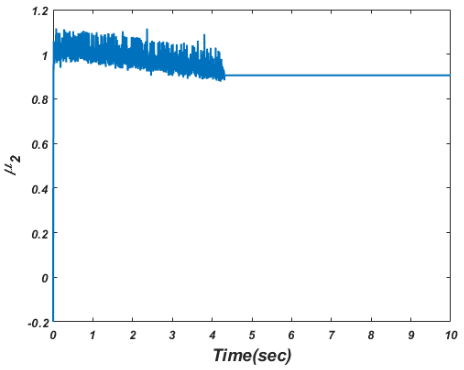
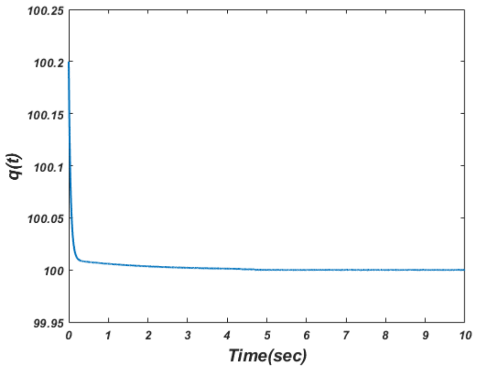
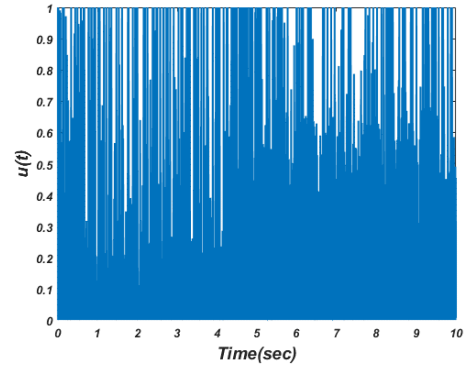


(d)

Fig. 5. The queue length (a) q(t), (b) control input u(t), adaptive parameters (c) $\hat{\mu}_1$ and (d) $\hat{\mu}_2$ for $\alpha_2 = 0.5$, $\beta_2 = 0.5$ and the external disturbance $\omega_1(t)$



(a)



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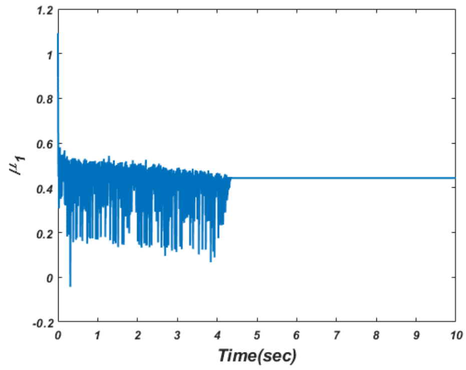

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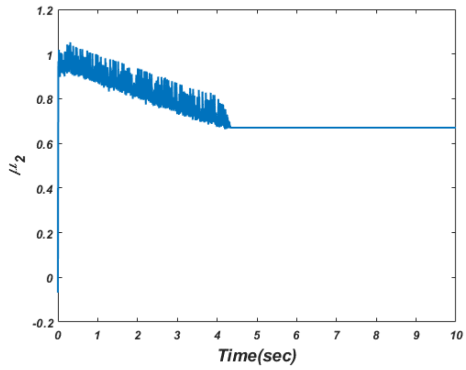
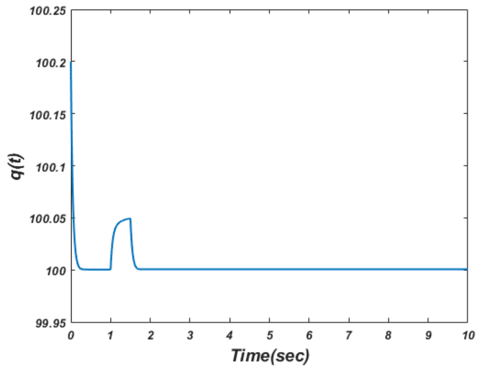
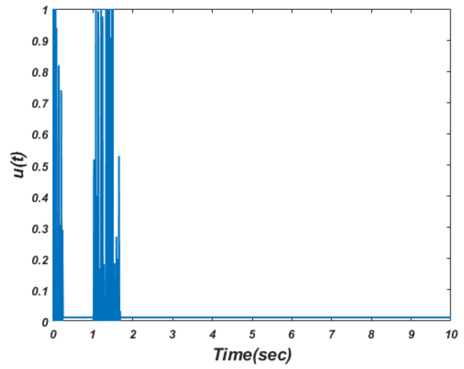


(d)

Fig. 6. The queue length (a) q(t), (b) control input u(t), adaptive parameters (c) $\hat{\mu}_1$ and (d) $\hat{\mu}_2$ for $\alpha_2 = 0.6$, $\beta_2 = 0.3$ and the external disturbance $\omega_1(t)$



(a)



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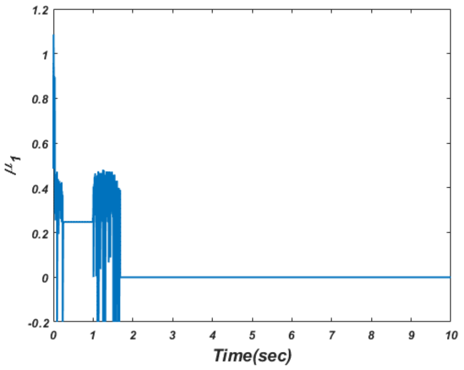

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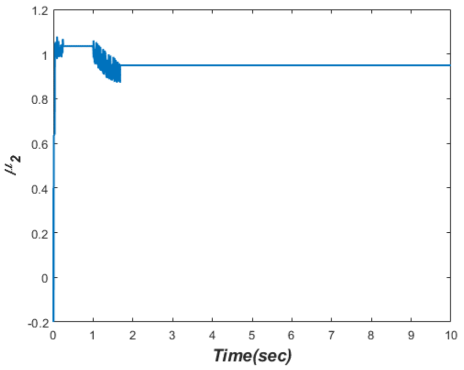

(d)

Fig. 7. The queue length (a) q(t), (b) control input u(t), adaptive parameters (c) $\hat{\mu}_1$ and (d) $\hat{\mu}_2$ for $\alpha_2 = 1,\ \beta_2 = 0$ and the external disturbance $\omega_2(t)$

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