Optimized Adaptive Combined Hierarchical Sliding Mode Controller Design for a Class of Uncertain Under-actuated Time-Varying Systems

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Abstract
This paper proposes an optimized adaptive combined hierarchical sliding mode controller (ACHSMC) for a class of under-actuated time-varying systems in presence of uncertainties and noise. For this purpose, the un-modeled dynamics and friction force are modeled as additive and multiplicative uncertainties, respectively. A combined hierarchical sliding mode controller (CHSMC) is designed using two layers of sliding manifolds. Then, the controller is adapted by considering a time-varying coefficient of the second layer sliding manifold of CHSMC system. The stability of this controller is approved by Lyapunov theorem. Finally, this method is performed on an under-actuated crane model that has two subsystems: trolley and payload can be controlled by a single input signal and the first layer sliding manifold parameter of ACHSMC is optimized by genetic algorithm (GA) to save energy of input signal. The simulation results show the stability and robust performance of the proposed controller against input noise and additive and multiplicative uncertainties and time varying parameters of the system compared to CHSMC method.

Keywords
Optimized adaptive controller, Combined hierarchical sliding mode controller, Under-actuated time varying system, Additive and multiplicative uncertainty, Genetic algorithm.

1. Introduction

In recent decades, many works have been presented to control of under-actuated nonlinear systems such as crane system [1], inertia-wheel pendulum [2-4] and etc. [5]. Also, fuzzy method has been applied to control under-actuated systems in both theoretical analysis and practical application reviewed in [6]. The efforts [7, 8] have designed the fuzzy controllers for nonlinear overhead crane systems with input delay, actuator saturation and dead-zone compensation and a fuzzy-tuned PID anti-swing controller has been studied for the crane in [9].

One of the robust designs is SMC that includes switching manifold [10-13]. In [14], a second-order sliding mode (SOSM) controller has controlled an overhead crane system affected by external perturbations. The two control approaches of SMC with nonlinear sliding manifold and vibration strain rate feedback (SRF) have been used for flexible spacecraft attitude control in research [15].

The SMC is insensitive to variety of system uncertainties, disturbances, unknown inputs and perturbations. This feature actually provides capability more than controller resistance and ensures asymptotic stability of the system by the Lyapunov theorem. The fractional-order SMC has been designed for nonlinear systems with uncertainty in [16-18]. The research [19] has controlled an isolated bridge with columns of irregular heights by a separated multilevel sliding mode controller (SMSMC) with three corresponding control signal. The research [20] has proposed a controller that is the combination of continuous terminal sliding mode control and adaptive control for a class of nonlinear systems in presence of perturbation. Nevertheless, a hierarchical SMC (HSMC) can be designed by a single control signal to describe degree of importance of each system state variable and it is suitable for under-actuated systems [21, 22]. In addition, an incremental HSMC approach has been proposed in [23]. To improve crane control performance, [24] has used sliding manifold with time-varying parameter. Design of adaptive SMC is a good choice for time-varying models. The work [25] has used a controller based on SMC by using Chebyshev neural network that its weights are tuned in real-time by using robust adaptive techniques. In [26], an adaptive model reference with HSMC has been developed for uncertain systems with time delay and dead-zone input. Qian et al. have presented an adaptive HSMC for the class of under-actuated systems in [27, 28].

In this paper, the definition of hierarchical sliding manifolds is considered as a separate linear combination of the main state variables and their derivatives for the system that makes it possible to easily evaluate between the main variables and the derivatives (i.e. system velocities) in the upper layer of ACHSMC system. The proposed definition of ACHSMC manifolds is a new method of controller design which is created as a combined process of state variables based on their importance in achieving design goals that at the same time, due to the time-varying nature of the system, an adaptive law is added to control algorithm. It is emphasized that this method is completely different from previous works that have used hierarchical incremental or aggregated methods. The purposed design method is presented for the under-actuated time-varying nonlinear systems that can overcome the complexity of such systems and satisfy the design objectives by designing the appropriate adaptive and switching control laws. Also, the model of uncertain mechanical systems is formulated in presence of uncertainty by modeling friction force and other un-modeled dynamics as multiplicative and additive uncertainties.

One of the advantages of this method is that the design purpose is achieved by adjusting fewer parameters in the sliding layers than other hierarchical methods, including aggregated and incremental approaches, due to the derivative relationship in the first layer of the sliding manifold of the ACHSMC method. In addition, this paper has the generality of the proposed method on a class of nonlinear systems, not just a specific system such as [23, 27, 28]. Stability analysis and achieving acceptable performance of the ACHSMC system requires significant skill and accuracy in controller design for the under-actuated time-varying nonlinear systems. In this work, the required designs of the switching and adaptive laws are done with several purposes, which are: (1) Ensuring the system stability, (2) reducing chattering, (3) system robustness and proper performance in the presence of existing uncertainties, and (4) optimization of control signal.

This paper is organized as follows: The dynamic of a group of mechanical systems is introduced and the frictional force and other uncertainties are modeled in section 2. In section 3, the combined HSMC is defined by intermediate variable. The proposed ACHSMC system is designed in section 4 and its stability is approved and optimized by the GA. The CHSMC method is given with the proof of stability in order to compare with the results of the optimized ACHSMC method in section 5. Finally, the simulation results for a crane system, the conclusions and recommendation are presented in section 6, 7 and 8.

2. Uncertainty modeling and problem formulation

The general form of a class of under-actuated mechanical systems is given by (1).

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f_1(X) + b_1(X)(u_{act}(t)) \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = f_2(X) + b_2(X)(u_{act}(t)) \\ \vdots \\ \dot{x}_{2n-1}(t) = x_{2n}(t) \\ \dot{x}_{2n}(t) = f_n(X) + b_n(X)(u_{act}(t)) \end{cases} \tag{1}$$

where $X(t) = [x_1(t)\; x_2(t) \ldots x_{2n}(t)]^T \in \Re^{2n}$ is the system state vector which is available, $u_{act}(t) \in \Re$ is the actuating force applied to the system, $f_i(X)$ and $b_i(X)$, $i = 1, 2, \ldots, n$ are unknown continuous nonlinear system functions.

2.1. Modeling of multiplicative frictional uncertainty

It is assumed that frictional force $u_{fric}(t)$ is applied to the system as uncertainty and so, the resultant force $u(t)$ defined in (2).

$$u(t) = u_{act}(t) - u_{fric}(t) \tag{2}$$

where $u_{fric}(t)$ is described in (3).

$$u_{fric}(t) = (\Delta b_i)u(t) \tag{3}$$

here Δb_i are the control gains of system. Therefore, the system state equations are rewritten as (4).

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f_1(X) + b_1(X)(1+\Delta b_1)u(t) \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = f_2(X) + b_2(X)(1+\Delta b_2)u(t) \\ \vdots \\ \dot{x}_{2n-1}(t) = x_{2n}(t) \\ \dot{x}_{2n}(t) = f_n(X) + b_n(X)(1+\Delta b_n)u(t) \end{cases} \tag{4}$$

The control gains Δb_i have the certain bounds, but there are unknown and modeled such as multiplicative uncertainty with input signal [10]. The boundaries are as $0 < \Delta b_{i_{\min}} \le \Delta b_i \le \Delta b_{i_{\max}}$. Therefore, the estimation of control gains $\widehat{\Delta b}_i$ is chosen as geometric means of boundaries is defined in (5).

$$\widehat{\Delta b}_i = (\Delta b_{i_{min}} \Delta b_{i_{max}})^{\frac{1}{2}} \tag{5}$$

Finally, the estimated nonlinear system functions $\hat{b}_{i0}(X)$ are obtained as (6).

$$\hat{b}_{i0}(X) = b_i(X)\left(1 + \widehat{\Delta b}_i\right) \tag{6}$$

2.2. Modeling of additive parametric uncertainty and noise

The nonlinear system functions p_i and n_i are additive uncertainty and input noise of the system as (7).

$$\left|\hat{f}_i - (p_i + n_i)\right| = \left|\hat{f}_i - f_i\right| \le F_i \tag{7}$$

where $\hat{f}_i$ are the average of the system dynamics and the uncertainty is limited by the known functions F_i.

3. Combined hierarchical sliding manifold description

The system state equations in (4) can be divided to two groups. One group is composed of the main state variables and the other group are the main variables derivatives respectively $[x_1, x_3, \ldots, x_{2n-1}]$ and $[x_2, x_4, \ldots, x_{2n}]$. Now, if it is assumed $X_d = [x_{1,d}\; x_{2,d}, \ldots, x_{2n,d}]^T$ is desired state vector, then matching errors can be considered for these groups as (8).

$$e_{2i-1} = x_{2i-1} - x_{2i-1,d}, \qquad e_{2i} = x_{2i} - x_{2i,d} \tag{8}$$

In order to design a combined hierarchical sliding manifold, first an intermediate variable z which is a linear combination of the matching errors of the main state variables is defined in first layer as (9).

$$z = \sum_{i=1}^{n} c_{2i-1} e_{2i-1} \tag{9}$$

where c_{2i-1} are the first layer coefficients. Also, the derivative of z that includes the matching errors of the second groups of states is expressed as (10).

$$\dot{z} = \sum_{i=1}^{n} c_{2i} e_{2i} \tag{10}$$

In the second layer, s is constructed by the intermediate variable and its derivative, which is determined by (11).

$$s = \alpha(t)z + \dot{z} \tag{11}$$

here $\alpha(t)$ is assumed the second layer time-varying parameter and can be positive or negative, so sliding manifold s could be in any quadrant in its phase plane. The schematic of two layer combined sliding manifold is illustrated in Fig. 1.

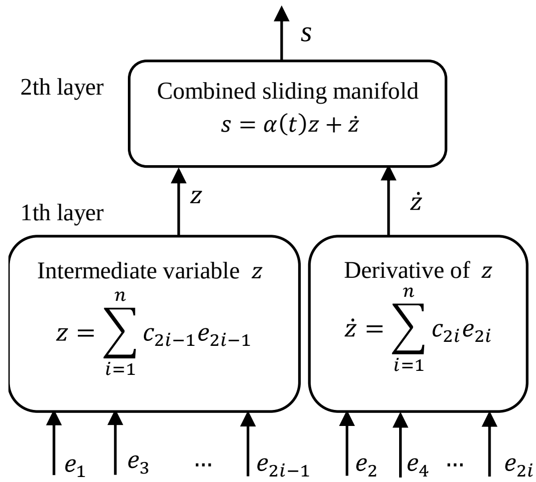

Fig. 1. Combined hierarchical sliding manifold.

4. Stability analysis and design of ACHSMC system

In this section, for stability of the system, a Lyapunov function candidate is defined in (12). By taking time derivative of V, $\dot{V}$ is obtained as (13).

$$V = \frac{s^2}{2} \tag{12}$$

$$\dot{V} = s\dot{s} = s(\dot{\alpha}(t)z + \alpha(t)\dot{z} + \ddot{z}) = s(\dot{\alpha}(t)z + \alpha(t)\textstyle\sum_{i=1}^{n} c_{2i}e_{2i} + \sum_{i=1}^{n} c_{2i}\dot{e}_{2i}) \tag{13}$$

By substituting (4) and (8) in (13), $\dot{V}$ is obtained as (14).

$$\begin{aligned}\dot{V} = s(\dot{\alpha}(t)z + \alpha(t)\sum_{i=1}^{n} c_{2i}e_{2i} + \sum_{i=1}^{n} c_{2i}f_i \\ + \sum_{i=1}^{n} c_{2i}b_i(1+\Delta b_i)(u_{sw}) \\ - \sum_{i=1}^{n} c_{2i}\dot{x}_{2i,d})\end{aligned} \tag{14}$$

Now, by considering the modeled multiplicative frictional uncertainty (5), the switching control law is defined as (15).

$$u_{sw} = -\frac{k\, sgn(s)}{\sum_{i=1}^{n} c_{2i}\hat{b}_{i0}(X)} \tag{15}$$

In (15), the discontinuous switching control law can cause chattering, leading to unpredictable instability, so it is used the smoothing function method in this paper to reduce chattering while trying to preserve good robustness properties around the sliding manifold. Therefore, the control law u_{sw} is obtained as (16).

$$u_{sw} = -\frac{ks}{(|s|+\varepsilon)(\sum_{i=1}^{n} c_{2i}\hat{b}_{i0}(X))} \tag{16}$$

here ε is a small positive constant. The adaptive parameter $\dot{\alpha}(t)$ is derived in (17) by assumption $\dot{s} = 0$ and considering the modeled additive parametric uncertainty (7).

$$\begin{aligned}\dot{\alpha} = -(\|z\|^2 + \delta)^{-1}(\alpha(t)\sum_{i=1}^{n} c_{2i}e_{2i} + \sum_{i=1}^{n} c_{2i}\hat{f}_i \\ - \sum_{i=1}^{n} c_{2i}\dot{x}_{2i,d})z\end{aligned} \tag{17}$$

δ is a small positive constant. Then, by Substituting (16) and (17) into (14), $\dot{V}$ is obtained as (18).

$$\begin{aligned}\dot{V} = s\left(\sum_{i=1}^{n} c_{2i}b_{i0}(X)\right) \times \\ \left(\left(\sum_{i=1}^{n} c_{2i}\hat{b}_{i0}(X)\right)^{-1}\left(-k\frac{s}{|s|+\varepsilon}\right)\right)\end{aligned} \tag{18}$$

By assumption k in (19), $\dot{V}$ is negative definite and asymptotic stability of this system is approved.

$$\begin{aligned}k \geq \left(\sum_{i=1}^{n} c_{2i}\hat{b}_{i0}(X)\right)\left(\sum_{i=1}^{n} c_{2i}b_{i0}(X)\right)^{-1} \\ \times \quad (F_1 + cF_2)\end{aligned} \tag{19}$$

Finally, the optimized value of c is derived for each mass transferring by the crane from initial point to destination point. For this purpose, the GA is used.

5. CHSMC method

If it is supposed that $\boldsymbol{\alpha(t) = \alpha}$ is a constant parameter in (11), this algorithm is not adaptive and it is CHSMC method. The combined control law is designed as (20) for the CHSMC method.

$$u = u_{eq} + u_{sw} \tag{20}$$

which u_{eq} is equivalent control law. To obtain the equivalent control law u_{eq}, the derivative of s is taken with respect to time t and the system model is substituted in it. Then, the law u_{eq} can be deduced from $\dot{s} = 0$ as (21).

$$u_{eq} = -\frac{\sum_{i=1}^{n} c_{2i}\hat{f}_i + \alpha\sum_{i=1}^{n} c_{2i}e_{2i}}{\sum_{i=1}^{n} c_{2i}b_{i0}(X)} \tag{21}$$

The stability of this method is proved by selecting Lyapunov function candidate (12). By substituting the control law (20) and the equivalent control law (21) in the derivative of $\boldsymbol{V}$, $\dot{\boldsymbol{V}}$ is achieved as (22).

$$\dot{V} = s(\alpha \sum_{i=1}^{n} c_{2i} e_{2i} + \sum_{i=1}^{n} c_{2i} f_i + \sum_{i=1}^{n} c_{2i} b_{i0}(X) \left(u_{sw} + u_{eq}\right) \quad (22)$$

For the purpose of system stability, the switching law $\boldsymbol{u_{sw}}$ is defined as follows:

$$u_{sw} = -\kappa s - \eta \frac{s}{|s| + \delta} \quad (23)$$

where κ, δ and η are positive constant coefficients.

6. Simulation results and discussions

In this section, the crane system in Fig. 2 is used to verify the performance of the proposed controller. Apparently, the system consists of two subsystems: trolley and payload. The payload is suspended from the trolley by a cable.

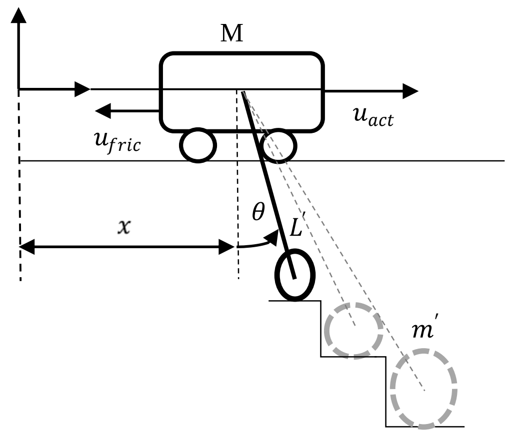

Fig. 2. The model of crane system.

Other symbols in Fig. 2 are described as the trolley mass M, the swing angle of the payload with respect to the vertical line θ, the trolley position x respect to the origin. $m'(t)$ and $L'(t)$ are time-varying payload mass and cable length that considered as (24) and (25).

$$m' = m + \frac{2M}{T} t \quad (24)$$

$$L' = L(1 + \frac{1}{T} t) \quad (25)$$

Equations (24) and (25) show time-variant parameters from $T = 0$ to 15 second and after of this time are fixed. Dynamic model of the crane system in the state space domain is considered in (26) from [20]. Here, $x_1 = x$, $x_3 = \theta$, x_2 is the trolley velocity; x_4 is the angular velocity of the payload.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(X) + b_{10}(X)u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(X) + b_{20}(X)u \end{cases} \quad (26)$$

Also, the estimation of dynamics $\hat{f}_1$ and $\hat{f}_2$ are as (27).

$$\hat{f}_1 = \frac{mL' x_4^2 \sin x_3 + m' g \sin x_3 \cos x_3}{M + m' \sin^2 x_3}$$
$$\hat{f}_2 = -\frac{(M + m') g \sin x_3 + ML' x_4^2 \sin x_3 \cos x_3}{(M + m' \sin^2 x_3)L'} \quad (27)$$

Furthermore, the nonlinear system functions b_1 and b_2 are defined as (28).

$$b_1 = \frac{1}{M + m' \sin^2 x_3}$$
$$b_2 = \frac{\cos x_3}{(M + m' \sin^2 x_3)L'} \quad (28)$$

The parameters of the crane model and initial and destination state vectors are given in Table I.

Table I. Physical parameters and desirable and initial vectors.

Parameters and Vectors	Value
Trolley mass M (kg)	1
Payload mass m (kg)	0.8
Cable length L (m)	0.305
Acceleration of gravity g ($\frac{m}{s^2}$)	9.81
Initial state vector x_0	$[0\,m \;\; 0\,m\,s^{-1} \;\; 0\,rad \;\; 0\,rad\,s^{-1}]$
Destination state vector x_d	$[1\,m \;\; 0\,m\,s^{-1} \;\; 0\,rad \;\; 0\,rad\,s^{-1}]$
Additive uncertainties	$F_1 = F_2 = 2$
Multiplicative uncertainties	$\Delta b_1 = \Delta b_2 = 0.1$
Weight of input white noise	0.01

By normalizing (9) and (10) and due to the fact that the first layer sliding coefficients are considered constant, the intermediate variable z and its derivative $\dot{z}$ are expressed as (29) and (30).

$$z = e_1 + ce_3 \quad (29)$$

$$\dot{z} = e_2 + ce_4 \quad (30)$$

The GA is used to optimize the energy of input control signal by specifying the constant variable c . The population size is 8 chromosomes. The crossover, mutation percentages and the selection pressure are relatively equal to 0.8, 0.3 and 10. The GA selection method can be chosen as roulette wheel or random selection in MATLAB software. By using GA algorithm for optimized ACHSMC, the value of c is derived as 0.02.

Finally, the optimized ACHSMC method is compared with CHSMC to verify suitable performance of the proposed controller. For this purpose, the coefficients c and $\alpha$of the two layers are considered equal to 0.242 and 0.487 in (21) for the CHSMC simulation. The time-varying parameter $\alpha(t)$ in the optimized ACHSMC is seen in Fig. 3. According to (9) the importance of the trolley position error is very greater than payload angle error, so effect of this selection, will be seen in Figs. 4 and 5.

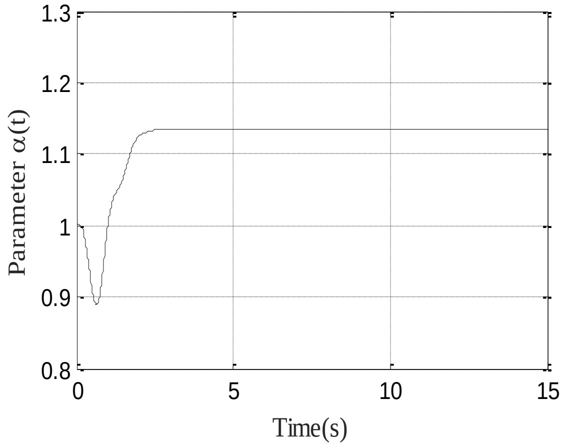

Fig. 3. Time-varying $\alpha(t)$ for ACHSMC.

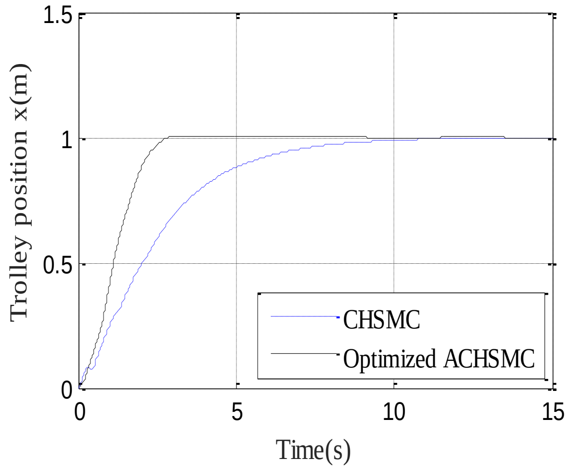

Fig. 4. Crane trolley position.

According to Fig. 4, the crane trolley reaches to destination point about 5 second, while this time for CHSMC is about 14 second. Fig. 5 shows although, the maximum payload angle deviation does not change much in the optimized ACHSMC method compared to the CHSMC method, but the payload angle becomes convergent to zero faster than CHSMC method.

In Fig. 6, the optimal input control signal u is smoothed and adapted by optimized ACHSMC and Fig. 7 shows the hierarchical sliding manifold of ACHSMC is very smoother rather than another method.

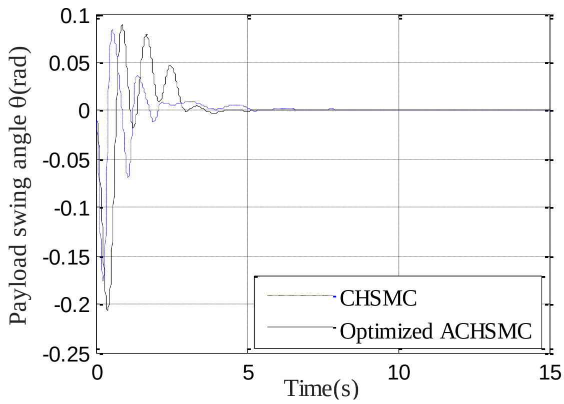

Fig. 5. Swing angle of payload θ.

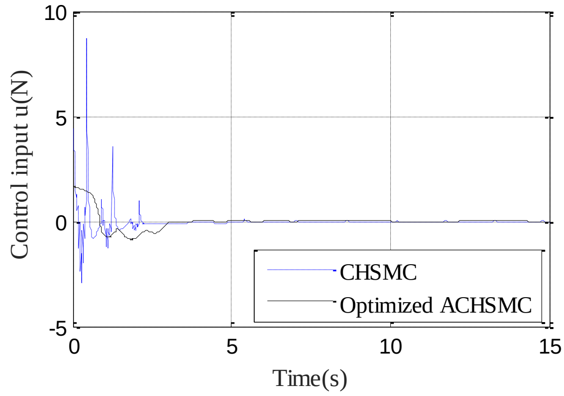

Fig. 6. Control signal.

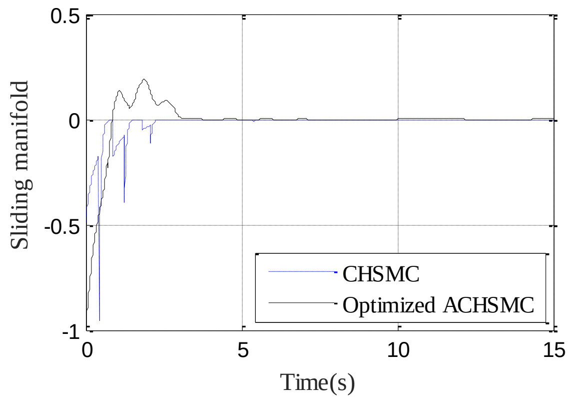

Fig. 7. Sliding manifold.

7. Conclusion

In this paper, an optimized robust controller was designed for a class of under-actuated time-varying systems in presence of uncertainties. The SMC is a good controller with robust performance. The un-modeled dynamics are modeled as additive uncertainties and friction force is modeled as multiplicative uncertainty. In the ACHSMC method first, a sliding surface was defined in first layer that was a linear combination of original state variables and another sliding surface was its derivative. Next, two surface were combined by a sliding manifold in second layer in the proposed method. For load transferring, its mass and cable length are time variant. Therefore, one of the innovations of the paper is the design of the adaptive and switching control laws in a way that ensures the system stability and the reduction of chattering.

By using the optimized ACHSMC approach, the purposed system has been able to maintain its robustness despite the existing additive and multiplicative uncertainties and the applied noise. The simulation of this controller was performed on the crane system to confirm the proper performance of the designed controller. Also, to save energy of input signal, the parameter of the first layer sliding manifold was optimized by genetic algorithm. The optimized ACHSMC method was compared with CHSMC by MATLAB simulation. The results showed the optimized ACHSMC method has better behaviors such as optimal signal control, smooth sliding manifold, and robust performance rather than CHSMC method.

8. Recommendation

Since in many cases, it is not possible or economical to measure all state variables in real applications, it is suggested to design an observer such that estimates the unmeasurable states of a system based only on the measured outputs and inputs and then, the proposed optimized ACHSMC method is designed based on the estimated states. Also, this method can be developed as optimized fractional order ACHSMC method to enhance control performance. To achieve this purpose, intelligent designs such as fuzzy neural networks are also efficient.

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